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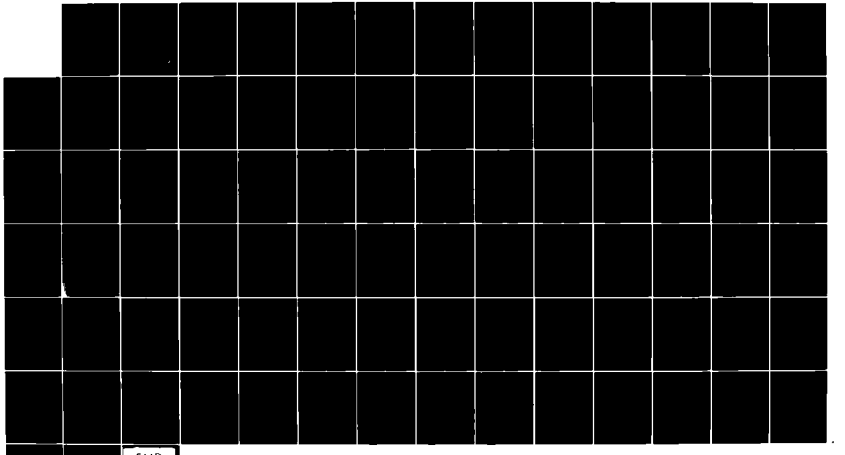

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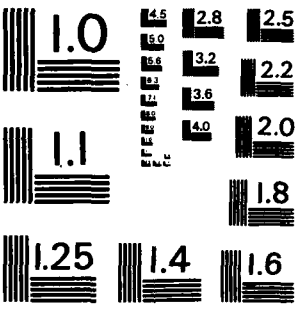

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REVISED COMPUTER PROGRAM FOR EVALUATING THE IVES TRANSFORMATION IN TURBOMACHINERY CASCADES

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20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

This report contains the description of a computer program for evaluating the Ives transformation, which maps a cascade of turbine or compressor blades conformally into a rectangle.

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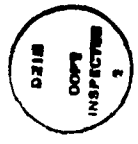
FOREWORD

This document supersedes the previous AFOSR Scientific Report cited below. The present report contains a number of improvements in the numerical methods used, and is applicable to a wider range of blade geometries. In addition, a number of errors in Ref. 1 have been corrected, and more complete descriptions are given for certain features of the method.

The author is very indebted to Joseph P. Nenni, John R. Moselle, and Marcia J. Williams for their assistance in the development of this program.



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1. Rae, W.J., "A Computer Program for the Ives Transformation in Turbomachinery Cascades", Calspan Report No. 6275-A-3, AFOSR TR-81-0154 (November 1980).

ABSTRACT

This report contains the description of a computer program for evaluating the Ives transformation, which maps a cascade of turbine or compressor blades conformally into a rectangle.

TABLE OF CONTENTS

Section 1
INTRODUCTION

Some of the most important recent advances in computational fluid dynamics have been made possible by the use of algorithms which solve the equations of motion in boundary-conforming coordinates. Thus the development of methods for carrying out these coordinate transformations has received considerable attention (see, for example, Reference 2). The methods in current use can be divided into three categories: those which apply algebraic shearing or stretching of the coordinates, those which are generated numerically by solving a Poisson equation, and those which are based on a conformal transformation.

This report is concerned with one example from the third category, namely the transformation introduced by Ives and Liutermoza.[3] It is capable of transforming a cascade of airfoils into a rectangle, thus facilitating the application of the solution algorithms mentioned above. The content of this report is a review of the transformation itself, together with a practical numerical procedure for applying it to a given cascade. This procedure involves a number of choices, involving branch-point locations, tolerances for various iteration sequences, and formulas for the evaluation of certain special functions.

Section 2 below contains a description of the transformation itself, including modifications that enable the mapping of blades having a rounded trailing edge, and the calculation of grids in which one of the coordinate lines joins the trailing edge to the point at downstream infinity.[4]

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2. Thompson, J.F., ed., Numerical Grid Generation, Elsevier Science Publishing Co., New York (1982).

3. Ives, D.C. and Liutermoza, J.F., "Analysis of Transonic Cascade Flow Using Conformal Mapping and Relaxation Techniques", AIAA Journal 15 (1977), pp. 647-652.

4. Rae, W.J., "Modifications of the Ives-Liutermoza Conformal-Mapping Procedure for Turbomachinery Cascades", ASME Paper 83-GT-116 (March 1983).

The final sections contain a description of the computer code and comments on its range of applicability.

Section 2
CONFORMAL TRANSFORMATION METHOD

The method of Ives & Liutermoza consists of a sequence of transformations, which map a two-dimensional cascade into a rectangle. The notation and coordinate system used to define the cascade are shown in Fig. 1. The x-coordinate is measured in the axial direction, and y is perpendicular to x.

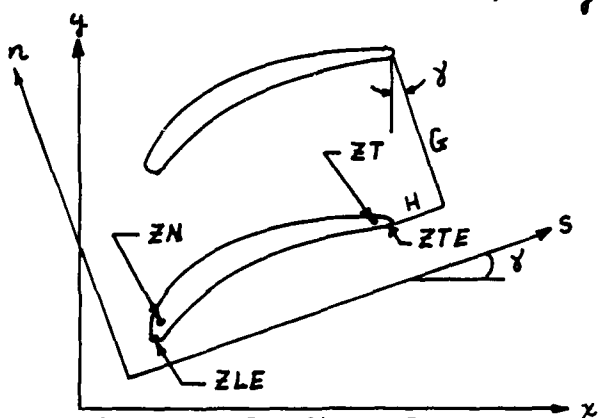


Figure 1. Coordinate System

The quantities s, n and the angle γ denote the "streamwise, normal" coordinates, in terms of which the blade profiles are sometimes defined. These reduce to the x, y set if H is taken as zero.* The origins of both of these coordinate systems are arbitrary.

These coordinates define a complex variable z :

$$z = s + in \tag{2-1}$$

The points ZN and ZT are taken anywhere near the centers of curvature of the leading and trailing edges, while ZLE and ZTE are points which divide the "pressure" side of the blade (i.e., its concave surface) from the "suction" side (its convex surface). These points can be chosen anywhere on the

*Actually, H must not be set identically equal to zero, but to the value $\pm 10^{-9}$, where the $\pm$ signs apply to compressors or turbines, respectively.

leading- and trailing-edge contours; ZTE is the point that will be connected to the "point" at downstream infinity by one of the grid lines, if that option is chosen (i.e., ISHEAR=1).

For the case of a sharp trailing edge (ITE=0), ZT must equal ZTE, and for a sharp leading edge (ILE=0), ZN must equal ZLE. The included angle at a sharp trailing edge, τ , must be specified. This is illustrated in Fig. 2, for the cascade used by Rae and Homicz.[5]

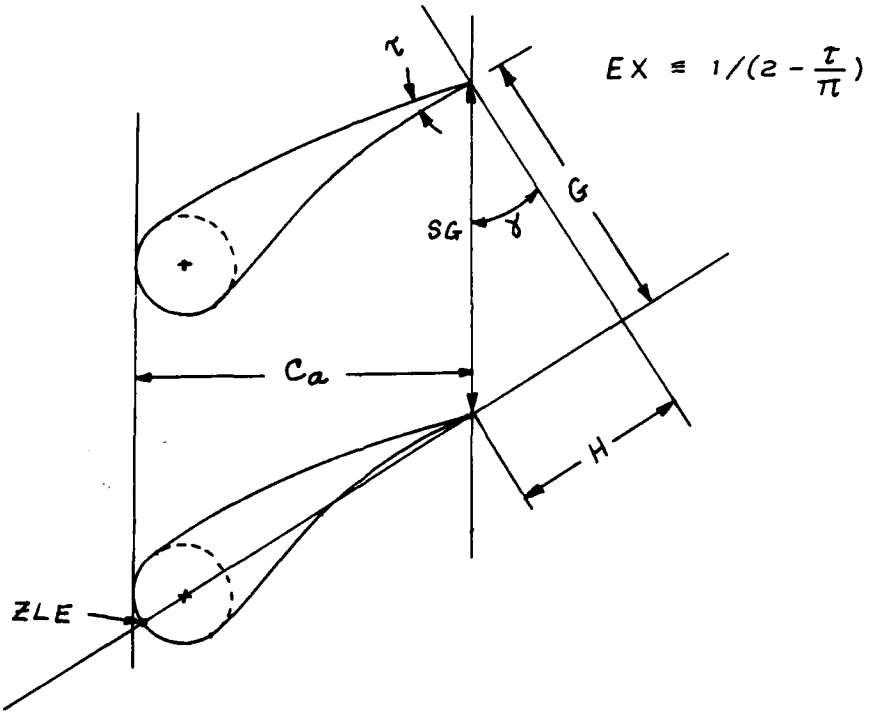


Figure 2. Blade Geometry of Ref. 5

This blade row has a slant-gap/axial chord ratio $SG/C_a = 1$, where SG is the slant gap:

5. Rae, W.J., and Homicz, G.F., "A Rectangular-Coordinate Method for Calculating Nonlinear Transonic Potential Flowfields in Compressor Cascades", AIAA Paper 78-248 (January 1978).

$$SG = \sqrt{H^2 + G^2} \tag{2-2}$$

and a stagger angle γ of 32.91°. This geometry is used, below, to illustrate the steps in the Ives transformation. A comparable set of illustrations, for a cascade of turbine blades with rounded trailing edges, is given in Ref. 4.

Figure 2 shows the relation between the trailing-edge angle τ and an exponent EX (which is used in one of the transformation steps described below). This relation applies only for a sharp trailing edge. For a rounded trailing edge, EX is not related to the 180-degree trailing-edge angle, but is chosen as a number in the range 0.2 to 0.4, as described below.

The blade shape is input as two tables of coordinate pairs, one for the pressure surface, and one for the suction surface. These coordinates, plus the leading- and trailing-edge points ZLE and ZTE are then arranged in an array indexed by KJ, where KJ=1 at the trailing edge, and where the numbering proceeds around the pressure side to the leading edge (KJLE), and then along the suction side to the trailing edge, where the point denoted by KJMX is a repeat of that denoted by KJ=1. This notation is shown in Fig. 3.

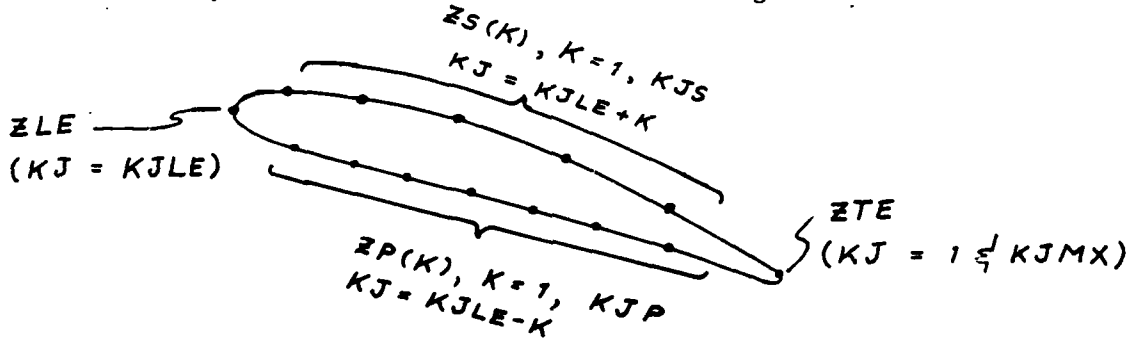


Figure 3. Notation for Blade-Surface Coordinates

The quantities KJS and KJP need not be equal; they are limited to a maximum value of 80 by a dimension statement in the current version of the program.

The first step in the Ives transformation is

$$q(z) = \frac{\sin\left\{\pi \dfrac{z - z_T}{H + iG}\right\}}{\sin\left\{\pi \dfrac{z - z_N}{H + iG}\right\}} \equiv \frac{\sin\zeta_1}{\sin\zeta_3} \equiv \frac{\zeta_2}{\zeta_4} \qquad (2\text{-}3)$$

The fact that only differences of z-values are used is what accounts for the arbitrariness of the origin in Fig. 1. On the suction (S) and pressure (P) sides, the function $q(z)$ has the Fortran equivalents:

$$q(z) = \begin{cases} RDS(K)\, e^{\,i\,THS(K)} \\ RDP(K)\, e^{\,i\,THP(K)} \end{cases} \qquad (2\text{-}4)$$

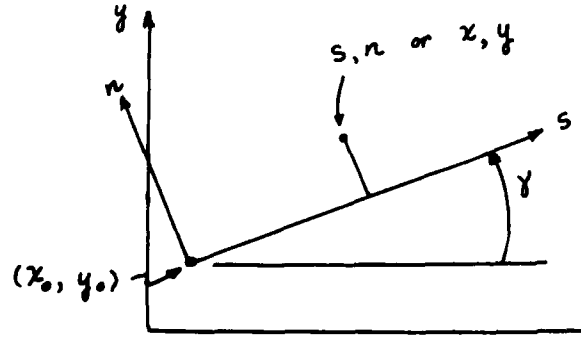
The arguments of the sine functions can be written in a simpler form, as follows:

$$\zeta_{1,3} = \pi \frac{\Delta z\,(H - iG)}{H^2 + G^2}$$

$$= \frac{\pi}{SG}\left\{Re\,\Delta z \cdot \sin\gamma + Im\,\Delta z \cdot \cos\gamma\right.$$

$$\left. + i\left[Im\,\Delta z \cdot \sin\gamma - Re\,\Delta z \cdot \cos\gamma\right]\right\} \qquad (2\text{-}5)$$

where Δz stands for $z - z_T$ or $z - z_N$ in the expressions for ζ_1 and ζ_3, respectively. By noting that (see Fig. 4)



$$x - x_0 = S\cos\gamma - n\sin\gamma$$

$$y - y_0 = S\sin\gamma + n\cos\gamma$$

Figure 4. Coordinate Relations

it follows that

$$\zeta_1 = \frac{\pi}{SG}\left\{(s - s_T)\sin\gamma + (n - n_T)\cos\gamma + i\left[(n - n_T)\sin\gamma - (s - s_T)\cos\gamma\right]\right\}$$

$$= \frac{\pi}{SG}\left\{y - y_T - i\,(x - x_T)\right\} \qquad (2\text{-}6)$$

Similarly,

$$\zeta_3 = \frac{\pi}{SG}\left\{y - y_N - i\,(x - x_N)\right\} \qquad (2\text{-}7)$$

The sine transformations:

$$\zeta_2 = \sin\zeta_1 \qquad \zeta_4 = \sin\zeta_3$$

map the strip $-\pi/2 \leq Re[\zeta_{1,3}] \leq +\pi/2$ into the entire ζ_2 (or ζ_4) plane, with cuts along the real axis from $-\infty$ to -1 and from +1 to $+\infty$:

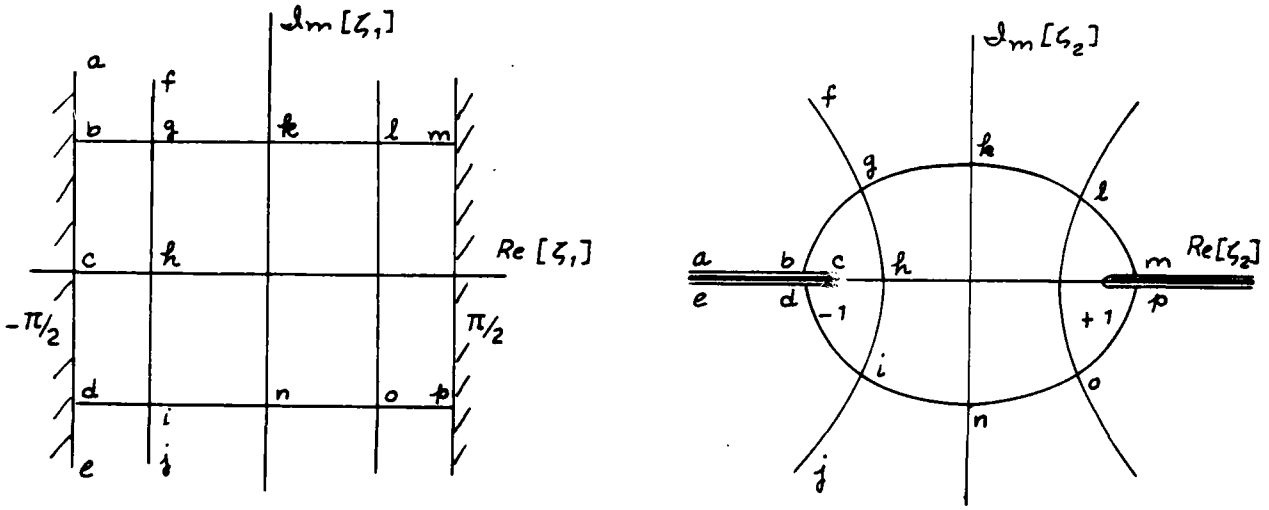


Figure 5. The Mapping $\zeta_2 = \sin\zeta_1$

In order to enforce this single-valuedness, the argument of the sine function should be displaced by the appropriate multiple of $H + iG$, whenever the real part of its argument falls outside the strip noted above, i.e., if $|Re\,\zeta_1| > \frac{\pi}{2}$, then set $\zeta = (\zeta + n\pi) - n\pi$, where n is chosen so as to make $|Re\,(\zeta + n\pi)| < \frac{\pi}{2}$. It turns out, however, that this displacement of the argument need not be made explicitly, when evaluating the complex sine. This can be seen by noting that

$$\sin(Re^{i\theta}) = \sin(R\cos\theta)\cosh(R\sin\theta) + i\cos(R\cos\theta)\sinh(R\sin\theta)$$

and that

$$\sin[(Re^{i\theta} + n\pi) - n\pi] = \cos n\pi \cdot \sin(R\cos\theta + n\pi + iR\sin\theta)$$

$$= \sin(R\cos\theta)\cosh(R\sin\theta) + i\cos(R\cos\theta)\sinh(R\sin\theta)$$

Thus, no special treatment is required for the argument of the complex sine. The result of this transformation is shown in Fig. 6, where S and P denote the suction and pressure sides. The quantity $q(\hat{z})$ is then formed from the ratio of the sines; this curve is shown in Fig. 7. The interior of the blade lies outside the closed curve in this plane. Because this function is to be raised to a power, its argument must be defined on the suction and pressure sides in such a way that the exponentiation will map the trailing-edge region into a straight segment. In the computer program, these arguments are first found from the Fortran DATAN2 function, which returns angles in the range from $-\pi$ to $+\pi$. These arguments are then adjusted, by defining the argument for KJ=2, and by then examining the points KJ=3 to KJMX, adding or subtracting 2π to the argument whenever the cut (along the negative real axis) in the DATAN2 function is traversed counterclockwise or clockwise, respectively. The arguments defined for KJ=2 use the following convention: for $H > 0$ (i.e., a stagger corresponding to a compressor blade row) the argument at KJ=2 is taken to be negative. For all other cases, the argument at KJ=1 is accepted as returned by the DATAN2 function.

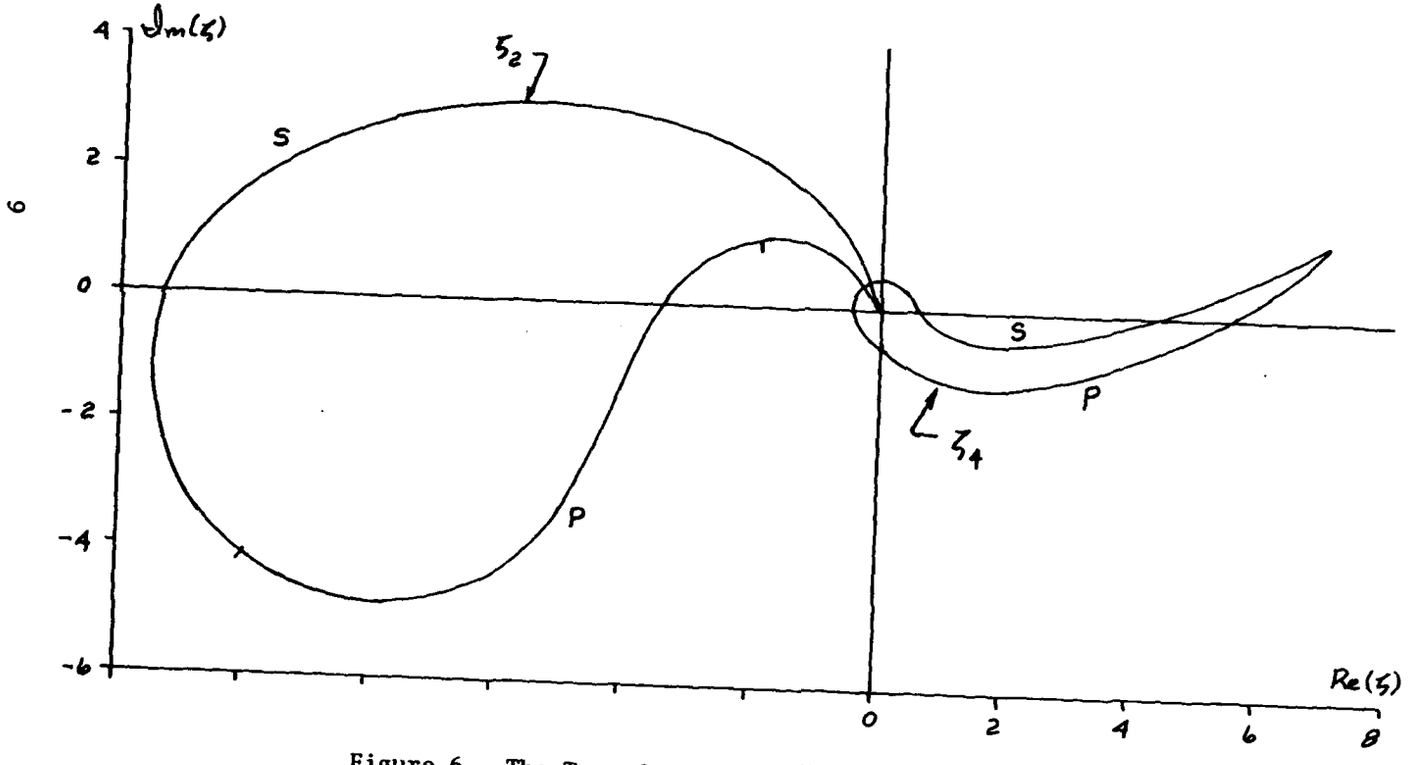


Figure 6. The Transformations ζ_2 and ζ_4

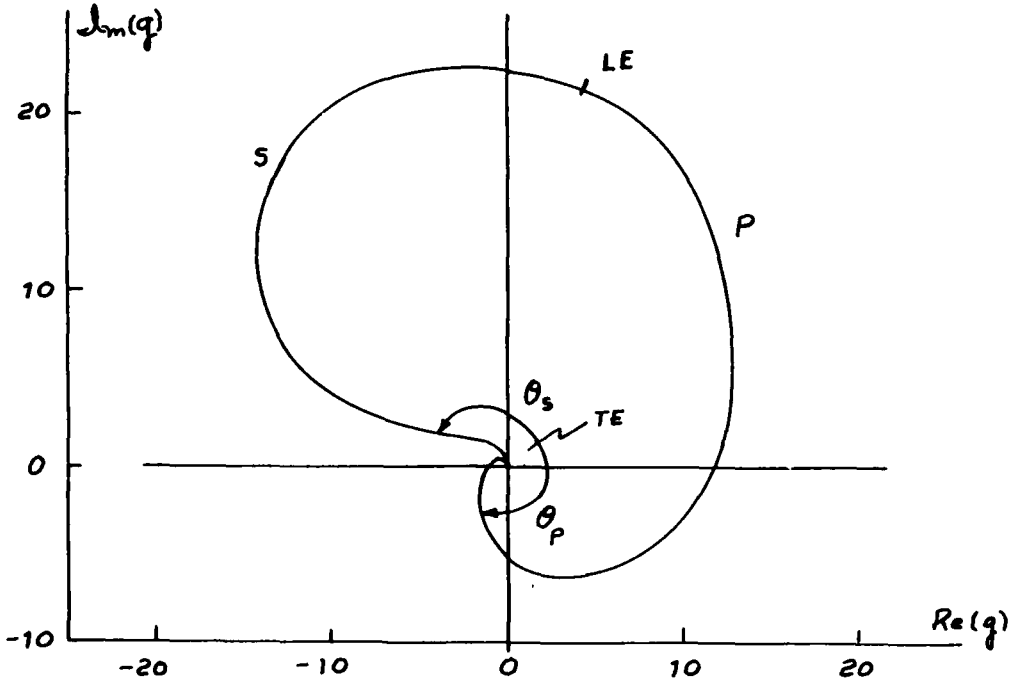


Figure 7. The Mapping $q(z)$

The next step is the exponentiation, defined as:

$$\Omega = [q(z)]^{1/K} \quad ; \quad K = 2 - \frac{\tau}{\pi} = EXINV \equiv \frac{1}{EX} \tag{2-8}$$

The result of this step is shown in Fig. 8. As noted above, the value of EX for a sharp-trailing-edge blade is fixed by the angle τ, whereas for a round trailing edge a range of values of EX, in the neighborhood of 0.3, will produce a curve in the Ω-plane that is "star-shaped", i.e., its polar coordinates will have a radius that is a single-valued function of the angular coordinate.

The next transformation is a bilinear one, whose purpose is to produce a curve in the ω-plane that can be mapped into a unit circle in a subsequent step:

$$C \frac{\omega - a}{\omega - b} = \Omega \quad ; \quad \omega = \frac{a - b\frac{\Omega}{C}}{1 - \frac{\Omega}{C}} \tag{2-9}$$

where a, b, and C are complex constants. Three conditions must be assigned, in order to evaluate these constants: Ives suggests, for two of them, that the images of upstream and downstream infinity be mapped into $\omega = \pm 1$. As the third condition, he recommends that the centroid of the blade-surface image in the ω-plane be forced to be close to the origin. This condition was applied in Ref. 1; however, it has been found simpler to impose a condition on the ratio between the maximum and minimum radii in the ω-plane, as outlined below. The calculation of the centroidal location has been retained in the present code, for informational purposes. (Details on how this is calculated can be found in Ref. 1).

The locations of the images of upstream and downstream infinity in the q- and Ω-planes can be expressed as follows: in general,

$$q(z) \equiv \frac{\sin\zeta_1}{\sin\zeta_3} = \frac{\sin(Re\,\zeta_1)\cosh(Im\,\zeta_1) + i\cos(Re\,\zeta_1)\sinh(Im\,\zeta_1)}{\sin(Re\,\zeta_3)\cosh(Im\,\zeta_3) + i\cos(Re\,\zeta_3)\sinh(Im\,\zeta_3)}$$

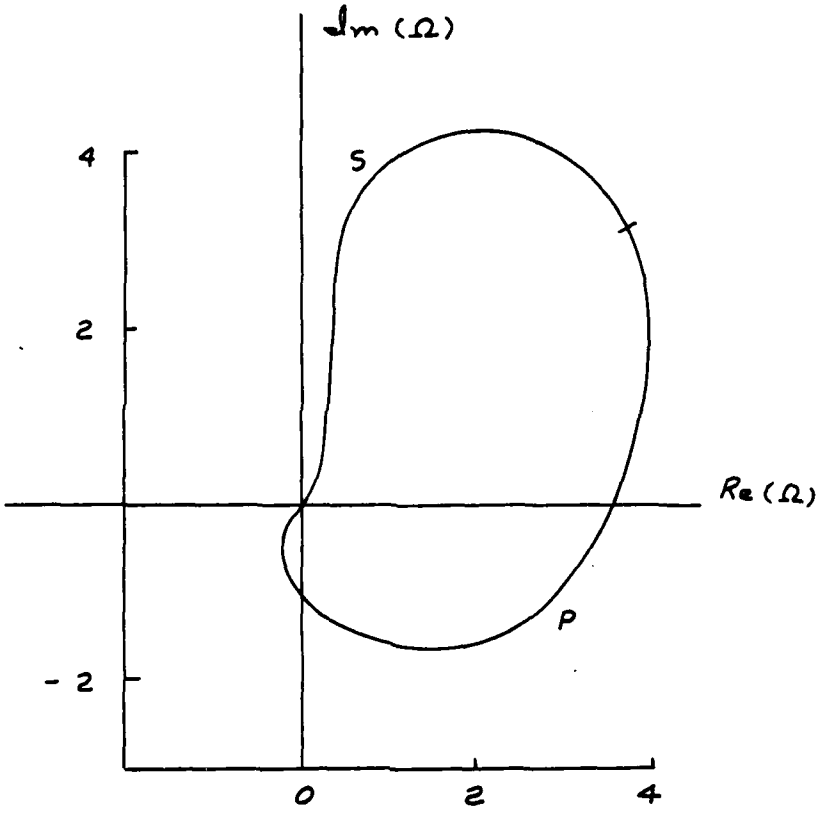


Figure 8. The Mapping $\Omega(z)$

As $z \to -\infty$ along line (1) of Fig. 9,

$$Re\,\zeta_1 = \text{finite},\quad Im\,\zeta_1 \to +\infty,\quad Re\,\zeta_3 = 0,\quad Im\,\zeta_3 \to +\infty$$

Using the large-argument approximations for the hyperbolic functions then gives

$$q(-\infty) = e^{Im(\zeta_1 - \zeta_3)}\, e^{-i\,Re\,\zeta_1} \equiv e^{\chi\cdot K}\, e^{i\chi_A\cdot K} \tag{2-10}$$

where a little algebra reveals that

$$\chi\cdot K \equiv Im(\zeta_1 - \zeta_3) = \frac{\pi}{SG}\left\{Re(z_T - z_N)\cos\gamma - Im(z_T - z_N)\sin\gamma\right\}$$

$$\chi_A\cdot K \equiv Re(\zeta_1) = -\frac{\pi}{SG}\left\{Re(z_T - z_N)\sin\gamma + Im(z_T - z_N)\cos\gamma\right\}$$

As $z \to +\infty$ along line (2) of Fig. 9,

$$Re\,\zeta_1 = 0,\quad Im\,\zeta_1 \to -\infty,\quad Re\,\zeta_3 = \text{finite},\quad Im\,\zeta_3 \to -\infty$$

and these lead to

$$q(+\infty) = e^{-Im(\zeta_1 - \zeta_3)}\, e^{-i\,Re(\zeta_3)}$$

where $Re(\zeta_3) = -Re(\zeta_1)$. Thus

$$q(+\infty) = e^{-\chi\cdot K}\, e^{-i\chi_A\cdot K} \tag{2-11}$$

and

$$\Omega^- \equiv [q(-\infty)]^{1/K} = e^{\chi} e^{i\chi_A},\quad \Omega^+ \equiv [q(+\infty)]^{1/K} = e^{-\chi} e^{-i\chi_A} \tag{2-12}$$

Note that

$$\Omega^+\,\Omega^- = 1$$

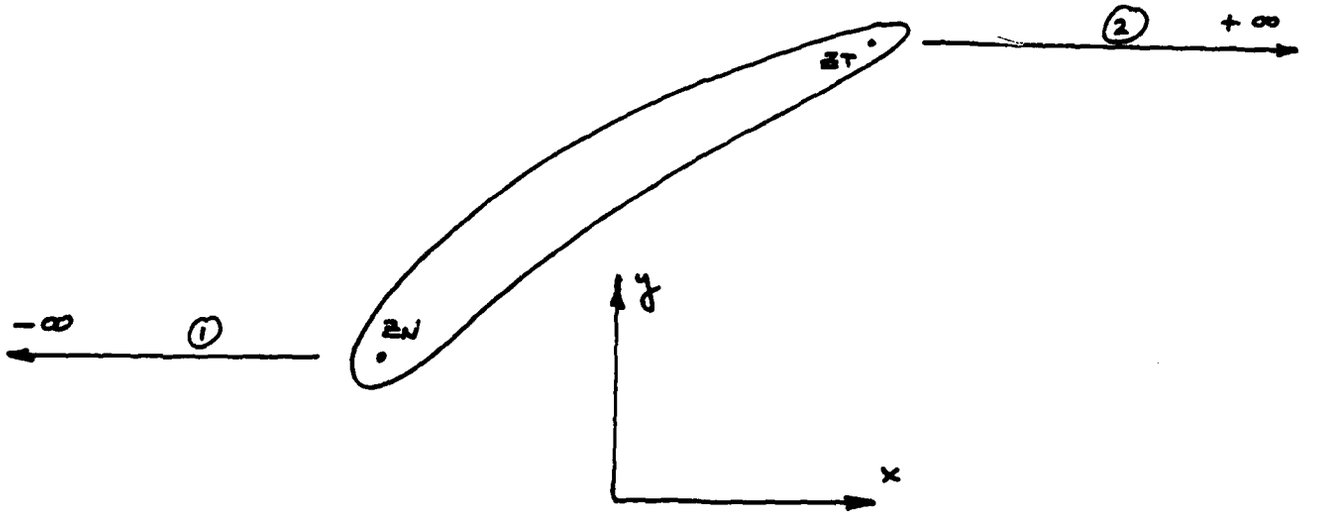


Figure 9. Location of the Points at Infinity

The constants a and b can be expressed in terms of C by the two equations:

$$C\,\frac{-1-a}{-1-b} \equiv \Omega^-$$

$$C\,\frac{1-a}{1-b} \equiv \Omega^+$$

The solution is:

$$b = \frac{-2C + \Omega^- + \Omega^+}{\Omega^+ - \Omega^-} \quad ; \quad a = 1 - (1-b)\,\frac{\Omega^+}{C}$$

$$= -Ec + F \quad ; \quad = \frac{E}{C} - F \qquad (2\text{-}13)$$

where E and F are known quantities:

$$E \equiv \frac{2}{\Omega^+ - \Omega^-} \quad , \quad F \equiv \frac{\Omega^+ + \Omega^-}{\Omega^+ - \Omega^-} \qquad (2\text{-}14)$$

In terms of the parameters E, F, and C, the transformation can be written as

$$\omega = \frac{E - Fc + (Ec - F)\,\Omega}{C - \Omega} \qquad (2\text{-}15)$$

$$\Omega = \frac{(\omega + F)c - E}{\omega - F + Ec} \qquad (2\text{-}16)$$

The latter relations can be used in an iteration procedure to find a value of C that will minimize the ratio RMAX/RMIN, where RMAX and RMIN are the maximum and minimum values of $|\omega|$ over the set defined by KJ=1 to KJMX. The iteration process is as follows: on alternate iterations, values of C are chosen that will either reduce RMAX or increase RMIN. This is done by solving Equation 2-16 for C :

$$C = \frac{\Omega\omega + E - F\Omega}{\omega + F - E\Omega} \qquad (2\text{-}17)$$

On the first iteration, $C = 1 + i$ is used; the values of RMIN and the index KJ=KJMN at which it occurs are then found. Next, a new value of C is calculated,

using Eq. 2-17, such that the new value of ω(KJMN) will equal 1.1 times the value just found. For this new mapping into the ω-plane, the value of RMAX and the index KJ=KJMXX at which it occurs are found. For the third iteration, a value of C is used such as to generate a new value of ω(KJMXX) equal to 0.9 times the previous one. This alternating cycle is then continued until the ratio RMAX/RMIN is less than the tolerance RTOL. This tolerance is assigned a default value of 3.0; values up to 6.0 have been handled successfully by the subsequent steps in the transformation.

The iteration on C can be bypassed, if C is already known, by setting IGOT=1 and reading in the value of C. Figure 10, reproduced from Ref. 1, was generated this way.

Next, the blade-surface image in the ω-plane is mapped into the unit circle in the ζ - plane with the trailing edge at $\zeta = 1$ by a variant of the Theodorsen-Garrick transformation:

$$\omega = \zeta \exp\left\{\sum_{j=0}^{N} (A_j + iB_j)\,\zeta^j\right\} \tag{2-18}$$

To determine the coefficients, the values of ω and ζ on the blade surface are written as

$$\omega = r(\theta)\,e^{i\theta}\,, \quad \zeta = 1\,e^{i\phi} \tag{2-19}$$

Then the real and imaginary parts of the transformation are

$$\ln r = A_0 + \sum_{j=1}^{N}\left[A_j \cos j\phi - B_j \sin j\phi\right] \tag{2-20}$$

$$\theta = \phi + B_0 + \sum_{j=1}^{N}\left[A_j \sin j\phi + B_j \cos j\phi\right] \tag{2-21}$$

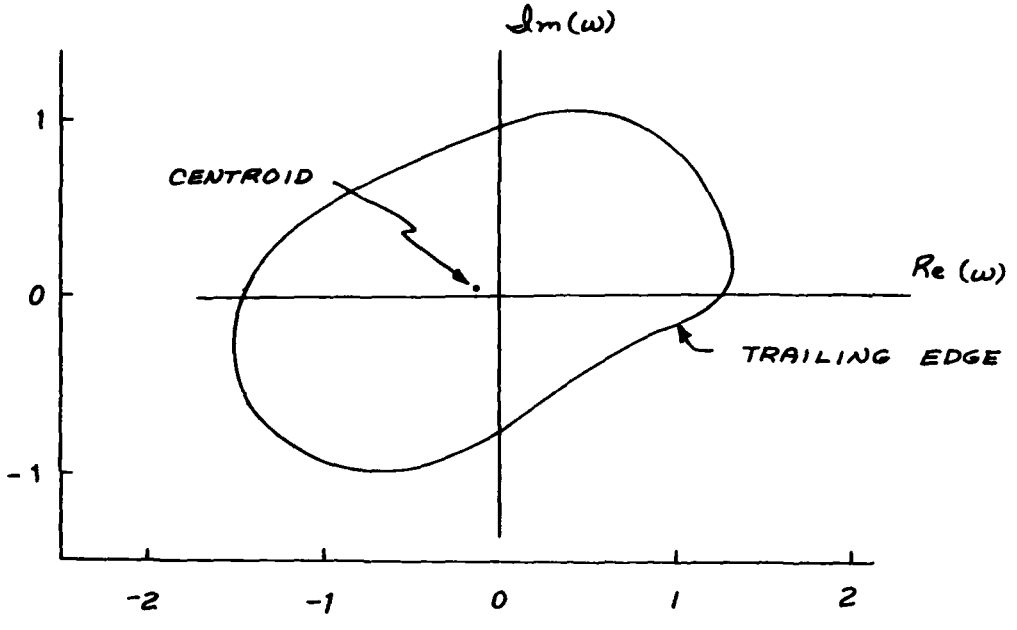


Figure 10. The Mapping $\omega(z)$

The coefficients are then determined by the following iteration procedure: an equally-spaced array of values of ϕ is set up, and all the coefficients are initially set equal to zero. Then the second equation gives $\theta = \phi$ as the first approximation for θ. For each of these values of θ, a corresponding value of $\ln r$ can be found, from a spline fit to the coordinates of the blade-surface image in the ω-plane. These known values of $\ln r$ are then used in the first of the equations above, to find the next approximation to the A_j and B_j coefficients. These coefficients can then be used to give the next approximation to $\theta(\phi)$, and the process is continued until convergence is reached, to some preassigned tolerance. Fast Fourier transform techtechniques[6] can be used in the processes of evaluating the second equation, for known values of the coefficients, and of determining the coefficients from the first equation with known values of $\ln r$. These techniques were applied in the present calculations. The details are given in Appendix A.

Sufficient conditions for convergence of this iteration process have been discussed by Warschawski.[7] For the present case, these conditions are not met; in particular, it is required that the maximum and minimum values of $r(\theta)$ obey the relation:

$$\sqrt{\frac{RMAX}{RMIN}} - 1 < 0.295 \qquad \text{or} \qquad \frac{RMAX}{RMIN} < 1.678$$

This condition is not met in the present case, where RMAX/RMIN is approximately 2.5. It was found, however, that the iteration process would converge

6. Cooley, J.W., Lewis, P.A.W., and Welch, P.D., "The Fast Fourier Transform Algorithm: Programming Considerations in the Calculations of Sine, Cosine and Laplace Transforms", Journal of Sound and Vibration 12, (1970) pp. 315-337.

7. Warschawski, S.E., "On Theodorsen's Method of Conformal Mapping of Nearly Circular Regions", Quarterly of Applied Mathematics 3, (1945) pp. 12-28.

if a relaxation parameter was used, i.e., the values of θ called for by the second equation [called θ^*] were not used in the first equation, but were replaced by $\theta_{new} = 0.1\,\theta^* + 0.9\,\theta_{old}$. With this relaxation factor, the iterations were convergent: after 68 iterations, the maximum change in any of the values of θ was less than 10^{-2} radians. The variation of θ with ϕ is shown in Fig. 11. Calculations for other cases, not shown here, have required relaxation factors as low as 0.02 for convergnece. A recent review paper by Henrici (Reference 8) calls attention to the applicability of under-relaxation in this problem.

The next transformation is

$$\eta = \gamma \frac{\zeta - \alpha}{\zeta - \beta} \tag{2-22}$$

where $\alpha, \beta,$ and γ are chosen so as to place the images of $z = \pm\infty$ at $\eta = \pm S$, while the blade surface continues to be the unit circle. These images are located, respectively, at $\omega = \pm 1$, and $\zeta = \zeta_A, \zeta_B$. Explicit formulas for α, β, and γ in terms of ζ_A and ζ_B are given by Ives[3] as

$$x = \frac{2 - |\zeta_A + \zeta_B|^2 + 2|\zeta_A \zeta_B|^2}{|\zeta_A - \zeta_B|^2}$$

$$S = \min\left\{\sqrt{|x + \sqrt{x^2-1}|}\ ,\ \sqrt{|x - \sqrt{x^2-1}|}\right\}$$

$$\alpha = \frac{2\zeta_A\zeta_B + [S^2(\zeta_A - \zeta_B) - (\zeta_A + \zeta_B)] / \tilde{\zeta}_A}{S^2(\zeta_A - \zeta_B) + (\zeta_A + \zeta_B) - 2/\tilde{\zeta}_A}$$

8. Henrici, P., "Fast Fourier Methods in Computational Complex Analysis". SIAM Review 21, (1979) pp. 481-527.

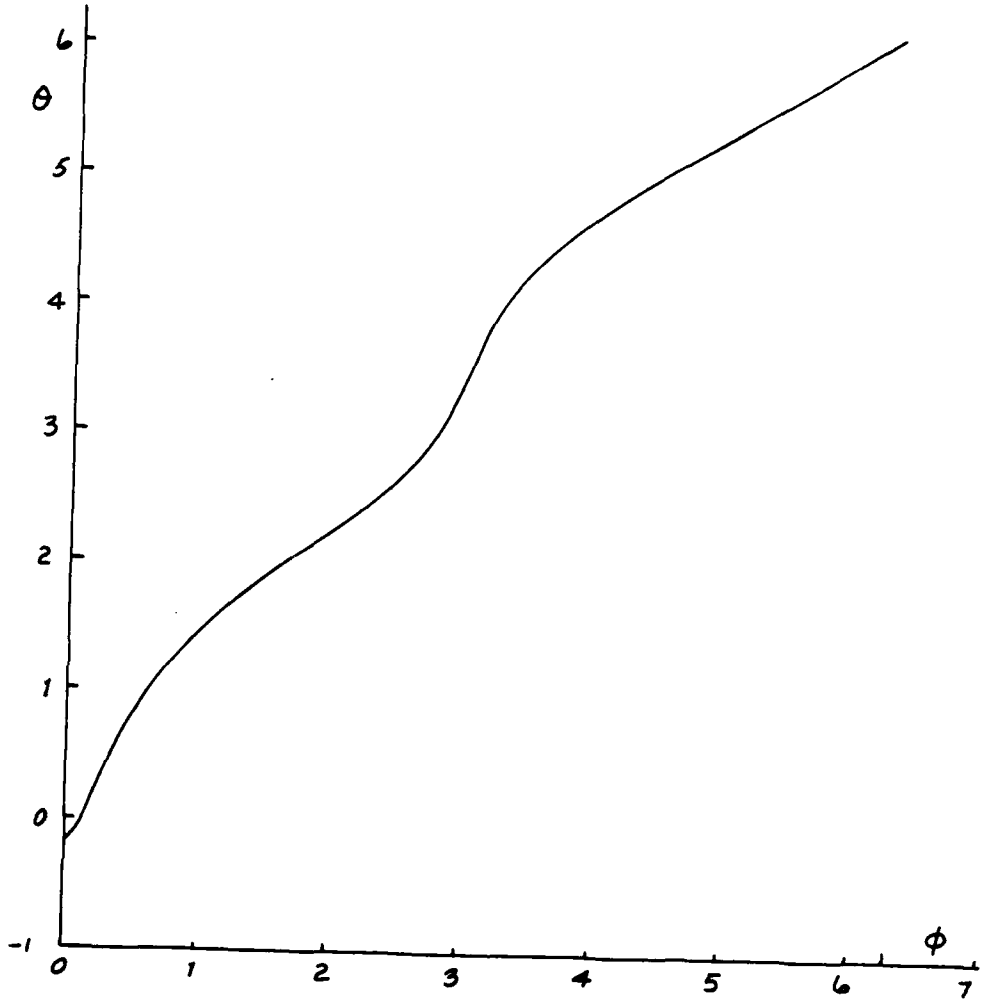


Figure 11. Variation of θ vs. ϕ on the Blade Surface

$$\beta = \frac{2\zeta_A\zeta_B - \alpha(\zeta_A + \zeta_B)}{\zeta_A + \zeta_B - 2\alpha}$$

$$\gamma = \mathcal{S}\,\frac{\zeta_A - \beta}{\zeta_A - \alpha} \qquad (2\text{-}23)$$

Mokry (Reference 9) has pointed out that the formula for $\mathcal{S}$ can be simplified, as follows: define

$$C \equiv \frac{\zeta_A\bar{\zeta}_B - 1}{\zeta_A - \zeta_B}$$

Then

$$\mathcal{S} = |C| - \sqrt{|C|^2 - 1} \quad , \quad \gamma = \frac{C - \mathcal{S}|C|\,\bar{\zeta}_B}{|C|\,\bar{\zeta}_B - \mathcal{S}C}$$

$$\alpha = \frac{\zeta_B C - \mathcal{S}|C|}{C - \mathcal{S}|C|\,\bar{\zeta}_B} \quad , \quad \beta = \frac{|C| - \mathcal{S}C\zeta_B}{|C|\,\bar{\zeta}_B - \mathcal{S}C} \qquad (2\text{-}24)$$

The computer program described below does not use these simplifications.

Next, it is necessary to find ζ_A and ζ_B, given $\omega = \pm 1$. This was done by Newton-Raphson iteration:

$$\zeta^{(n+1)} = \zeta^{(n)} - \frac{G(\zeta)}{\frac{dG}{d\zeta}}$$

---

9. Mokry, M., "Comment on Analysis of Transonic Cascade Flow Using Conformal Mapping and Relaxation Techniques", AIAA Journal 16, No. 1, (January 1978) p. 96.

where

$$G(\zeta) = \zeta \exp\left\{\sum_{j=0}^{N} (A_j + iB_j)\,\zeta^j\right\} - \omega$$

$$\frac{dG}{d\zeta} = \frac{\omega}{\zeta}\left[1 + \sum_{j=1}^{N} j\,(A_j + iB_j)\,\zeta^j\right]$$

In order to find an initial guess for ζ, a preliminary calculation was made, for

$$|\zeta| = 0.49\ (.1)\ 0.99, \qquad \arg\zeta = -60°\ (10°)\ +60°$$

From this set, the value of ζ which gave ω nearest to +1 was chosen as the initial guess. This set was then repeated with ζ replaced by $-\zeta$, to obtain the value of ζ that gave ω nearest to -1. The locations of key points in the ζ and η planes are shown in Fig. 12.

When the values of ζ_A and ζ_B are known, the blade-surface-image points can be mapped from the ζ-plane to the η-plane. In both planes, these points are located on unit circles, so it is only necessary to interpolate in Fig. 12 to find the ζ-values for the points defining the blade surface. This is done with a call to the spline-fit subroutine, and the values returned by it are replaced by linear interpolation in regions where the Θ, Φ curve is so steep that the spline fit returns non-monotonic values.

The final transformation now uses an elliptic function to map the unit circle in the η-plane (with cuts along the real axis from $\pm S$ to the circle) into a rectangle:

$$\eta = S\,Sn(\tilde{\xi}, k) \tag{2-25}$$

where the parameter k is given by

$$k = S^2 \tag{2-26}$$

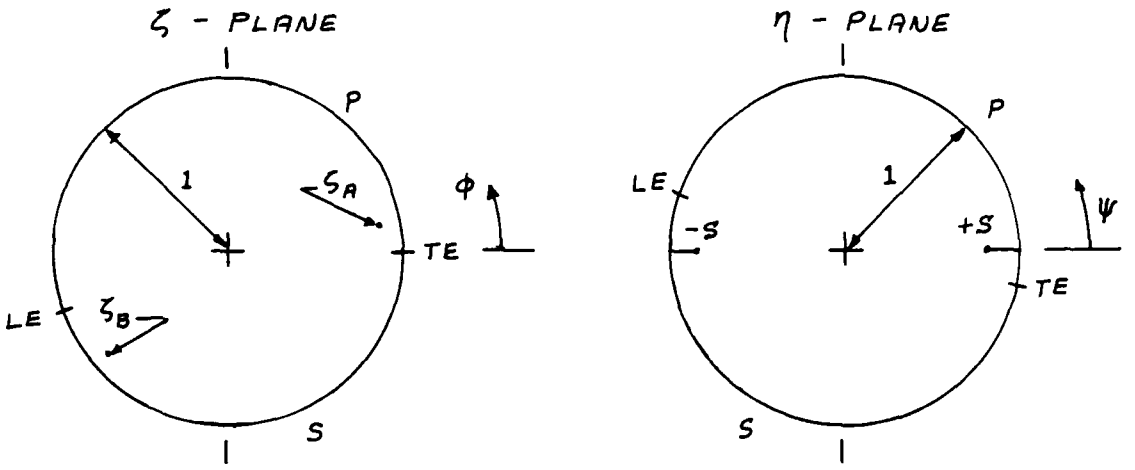


Figure 12. The ζ and η Planes

The inverse of this transformation is

$$\tilde{\xi} = \int_0^{\eta/S} \frac{dt}{\sqrt{1-t^2}\ \sqrt{1-k^2t^2}} \qquad (2\text{-}27)$$

This latter transformation is used to find the images, in the $\tilde{\xi}$ -plane, of the blade-surface points. This requires an expression for the real and imaginary parts of the incomplete elliptic integral of the first kind; convenient formulae for this purpose are given by Nielsen and Perkins[10] who show that, if

$$\eta = S(\tau + i\delta) \qquad (2\text{-}28)$$

then

$$\tilde{\xi} = F(\sqrt{\lambda}, k) + i F(\sqrt{\sigma}, k') \qquad (2\text{-}29)$$

where

$$k' = \sqrt{1-k^2} = \sqrt{1-S^4} \qquad (2\text{-}30)$$

10. Nielsen, J.N. and Perkins, E.W., "Charts for the Conical Part of the Downwash Field of Swept Wings at Supersonic Speeds", NACA Technical Note 1780, (December 1948), Appendix C.

and where F (· · ·) denotes the incomplete elliptic integral of the first kind, with real arguments λ and σ given as functions of τ, δ and k :

$$\lambda = \left[1+\tau^2+\delta^2 - \sqrt{(1-\tau^2)^2+\delta^2(\delta^2+2+2\tau^2)}\,\right]\left[1+k^2(\tau^2+\delta^2)\right.$$

$$\left. - \sqrt{(1-k^2\tau^2)^2+k^2\delta^2(2+2k^2\tau^2+k^2\delta^2)}\,\right] \Big/ 4k^2\tau^2$$

$$\sigma = \left[\tau^2+\delta^2-\lambda\right] \Big/ \left[\tau^2+\delta^2-\lambda+1-\lambda k^2(\tau^2+\delta^2)\right] \qquad (2\text{-}31)$$

These formulae are equivalent to those following Eqn. 115.01 of Byrd and Friedman;[11] the formula for λ has been rearranged slightly from the form given by Nielsen and Perkins, to avoid the occurence of negative values under the square root sign, which can sometimes happen in the numerical evaluations when $\delta = 0$. These formulas are correct along the branch cuts, where $\delta = 0$ and $|\eta|/\delta > 1$.

Numerical evaluations of these elliptic integrals were done using twelve terms in the formulae of Luke:[12]

$$F(y,k) \equiv \int_0^y \frac{dt}{\sqrt{1-t^2}\,\sqrt{1-k^2t^2}} = \int_0^{\phi=\sin^{-1}y} \frac{d\psi}{\sqrt{1-k^2\sin^2\psi}}$$

$$\approx F_{12}(\phi,k) = \frac{1}{25}\left[\phi + 2\sum_{m=1}^{12} \frac{\tan^{-1}(\sigma_m \tan\phi)}{\sigma_m}\right] \qquad (2\text{-}32)$$

11. Byrd, P.F., and Friedman, M.D., Handbook of Elliptic Integrals for Engineers and Physicists. Springer Verlag, Berlin (1954).

12. Luke, Y.L., "Approximations for Elliptic Integrals", Mathematics of Computation, 22 (1968) 627-634.

where

$$\phi < \pi/2 \ , \quad \sigma_m = \sqrt{1 - k^2 \sin^2 \theta_m} \ , \quad \theta_m = m\pi/25$$

For the case where $\phi = \pi/2$ (the complete integral) the approximate formula is

$$F_{12}\left(\frac{\pi}{2} \ , k\right) = \frac{\pi}{50}\left[1 + 2\sum_{m=1}^{12} \frac{1}{\sigma_m}\right] \tag{2-33}$$

The signs in the formulas above pertain to the first quadrant in the η -plane ($\tau \geq 0$, $\delta \geq 0$), which maps into a rectangle in the first quadrant of the $\tilde{\xi}$ -plane, with sides located on the lines

$$Re(\tilde{\xi}) = K(k) = \int_0^1 \frac{dt}{\sqrt{1-t^2}\sqrt{1-S^4 t^2}}$$

$$\mathcal{I}m(\tilde{\xi}) = \frac{1}{2}K'(k) = \frac{1}{2}\int_0^1 \frac{dt}{\sqrt{1-t^2}\sqrt{1-(1-S^4)\,t^2}} \tag{2-34}$$

The remaining three quadrants in the η -plane map into the remaining three quadrants in the $\tilde{\xi}$ -plane, as described in Reference (13), p. 377; the cuts from $\pm S$ to the unit circle along the real axis become the left and right sides of the rectangle in the $\tilde{\xi}$ -plane:

13. Erdelyi, A., et al. Higher Transcendental Functions, Volume 2, p. 377, McGraw-Hill Book Company, New York (1953).

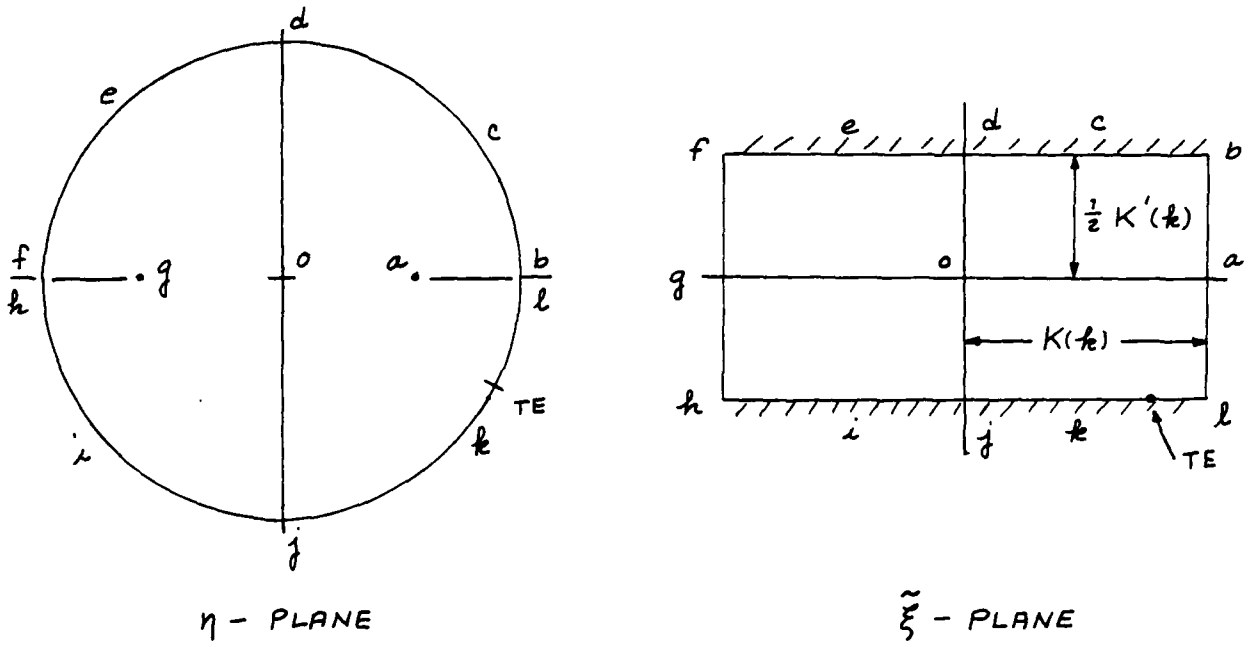


Figure 13. The η and $\tilde{\xi}$ Planes

Finally, the $\tilde{\xi}$ - plane is re-normalized, so as to lie between -1 and +1 on both axes:

$$z_{MAPPED} \equiv \frac{Re(\tilde{\xi})}{K(k)} + i\,\frac{\mathcal{I}_m(\tilde{\xi})}{\frac{1}{2}K'(k)} \qquad (2\text{-}35)$$

A grid is now to be set up in the $\tilde{\xi}$ plane, and mapped back to the z - plane. This process is facilitated by first rearranging the quadrants in the $\tilde{\xi}$ - plane, by using the periodicity of the elliptic functions as follows: in the first and second quadrants, let $\hat{\xi} = \tilde{\xi}$, and in the third and fourth let $\hat{\xi} = -(\tilde{\xi} + 2K(k))$ and use the relations (see for example, Eqs. 122.00 and 122.04 of Reference 11:)

$$Sn(\tilde{\xi}) = -Sn(\tilde{\xi}+2K) = Sn\left[-(\tilde{\xi}+2K)\right] = Sn\left[-K-(\tilde{\xi}+K)\right] \quad (2\text{-}36)$$

The $\hat{\xi}$ plane then has the form:

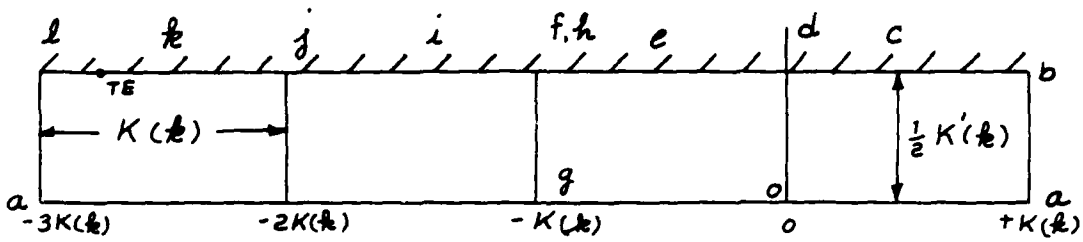


Figure 14. The $\hat{\xi}$ Plane

Two types of grid can be selected in the $\hat{\xi}$ plane: a rectangular one (if ISHEAR=0) or a sheared one (if ISHEAR=1). The latter is the default.

For the rectangular grid, equally spaced points are assigned, according to

$$\hat{\xi}_R \equiv Re(\hat{\xi}) = (K-1)\,\Delta\hat{\xi}_{real}\,, \quad K = 1,2,\ldots,KMX \quad (2\text{-}37)$$

$$\hat{\xi}_I \equiv Im(\hat{\xi}) = \tfrac{1}{2}K'(k) - (L-1)\,\Delta\hat{\xi}_{imag}\,, \quad L = 1,2,\ldots,LMX$$

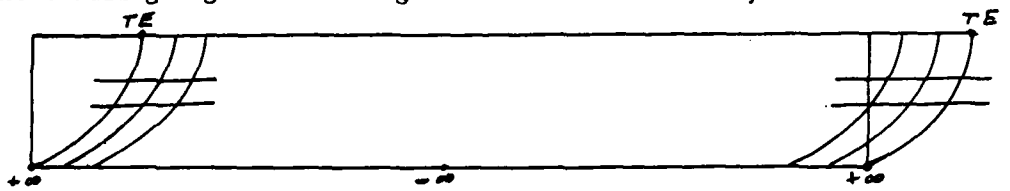
where

$$\Delta\hat{\xi}_{real} = \frac{4K(k)}{KMX-1}\,, \quad \Delta\hat{\xi}_{imag} = \frac{\frac{1}{2}K'(k)}{LMX-1} \quad (2\text{-}38)$$

Becuase the mapping is conformal, the images of these grid lines will intersect at right angles when mapped back to the physical plane.

The rectangular grid has the property that in general the trailing edge is not connected, by a grid line, to the point at downstream infinity. However, such a connection is a desirable feature in certain flowfield codes

(see Ref. 14, for example). To allow this feature, the ISHEAR=1 option establishes a sheared grid, consisting of the same $Im(\hat{\xi})$ lines as above, but replacing the $Re(\hat{\xi})$ lines by a set of parabolas which intersect the blade surface at 90 degrees, and are displaced from a base parabola that connects the trailing edge to the image of downstream infinity:



This grid is given by

$$\hat{\xi}_I = \tfrac{1}{2} K'(k) - (L-1)\,\Delta\hat{\xi}_{imag} \quad , \quad L = 1, 2, \ldots, LMX$$

$$\hat{\xi}_R = \hat{\xi}_R(TE) + (K-1)\,\Delta\hat{\xi}_{real} + \frac{\left[\hat{\xi}_I - \tfrac{1}{2}K'(k)\right]^2}{const} \quad , \quad \begin{matrix} K = 1, 2, \ldots, KMX \\ L = 1, 2, \ldots, LMX \end{matrix} \qquad (2\text{-}39)$$

where

$$const = \frac{-\left[K'(k)\right]^2}{4\left[3K(k) + \hat{\xi}_R(TE)\right]} \quad ; \quad \hat{\xi}_R(TE) = Re\left\{\hat{\xi}\left[KJ = 1\right]\right\} \qquad (2\text{-}40)$$

Some points on these parabolas will lie outside the range

$$-3K(k) \leq \hat{\xi}_R \leq +K(k)$$

When this occurs, equivalent points are found by adding or subtracting $4K(k)$, the period of the elliptic sine. In addition, the base parabola is always joined to the lower-left corner of Fig. 15, by subtracting $4K(k)$ from the real part of $\hat{\xi}_{TE}$, if the latter is greater than $-K(k)$.

This completes the definition of the grid in the $\hat{\xi}$-plane. Each of these grid points must now be mapped back to the physical plane. The first

14. Nenni, J.P. and Rae, W.J., "Experience with the Development of an Euler Code for Rotor Rows", ASME Paper 83-GT-36 (March 1983).

transformation is: $$\eta = S\, sn(\tilde{\xi}\,;\, k) = S\, sn(\xi\,;\, k) \qquad (2\text{-}41)$$

The elliptic sine of a complex argument is expressed as (Ref. 11, Eq. 125.01)

$$sn(u + iv, k)$$

$$= \frac{sn(u,k)\, dn(v,k') + i\, cn(u,k)\, dn(u,k)\, sn(v,k')\, cn(v,k')}{1 - sn^2(v,k')\, dn^2(u,k)} \qquad (2\text{-}42)$$

The functions in this expression are evaluated by the Arithmetic-Geometric Mean method (see Ref. 15, p. 571):

Set

$$a_0 = 1 \quad , \quad b_0 = k' \quad , \quad c_0 = k$$

and then calculate

$$a_n = \frac{1}{2}(a_{n-1} + b_{n-1}) \quad , \quad b_n = \sqrt{a_{n-1}\, b_{n-1}}$$

$$c_n = \frac{1}{2}(a_{n-1} - b_{n-1}) \qquad (2\text{-}43)$$

until $c_n = 0$ to a prescribed tolerance (10^{-7} was used in the present case.) Then form

$$\varphi_N = 2^N \cdot a_n \cdot u$$

and calculate φ_{N-1} , φ_{N-2} , $\cdots$, φ_0 from

$$\varphi_{n-1} = \frac{1}{2}\left\{\varphi_n + \arcsin\left(\frac{c_n}{a_n}\sin\varphi_n\right)\right\} \qquad (2\text{-}44)$$

Then the desired results are given by

---

15. Abramowitz, M. and Stegun, I.A., Handbook of Mathematical Functions, National Bureau of Standards, Applied Mathematics Series 55 (1964).

$$sn(u,k) = \sin\varphi_0 \quad , \quad cn(u,k) = \cos\varphi_0 \quad ,$$

$$dn(u,k) = \frac{\cos\varphi_0}{\cos(\varphi_1 - \varphi_0)} \tag{2-45}$$

These values of η are then mapped back to the ζ - plane by

$$\zeta = \frac{\beta\eta - \alpha\gamma}{\eta - \gamma} \tag{2-46}$$

and thence to the ω- plane by

$$\omega = \zeta \exp\left[\sum_{j=0}^{N} (A_j + iB_j)\,\zeta^j\right] \tag{2-47}$$

Lastly, the value of Z must be found by inverting:

$$\Omega \equiv c\,\frac{\omega - a}{\omega - b} = \left\{\frac{\sin\pi\,\dfrac{Z - Z_T}{H + iG}}{\sin\pi\,\dfrac{Z - Z_N}{H + iG}}\right\}^{1/K} \equiv [g(Z)]^{1/K} \tag{2-48}$$

This is written as

$$F(Z) = g(Z) - \Omega^K \tag{2-49}$$

and is solved by a Newton-Raphson procedure

$$Z^{(n+1)} = Z^{(n)} - \frac{F(Z^{(n)})}{F'(Z^{(n)})} \tag{2-50}$$

where

$$F'(Z) = \frac{\pi}{H + iG}\,\frac{\sin\dfrac{\pi(Z_T - Z_N)}{H + iG}}{\sin^2\pi\,\dfrac{Z - Z_N}{H + iG}} \tag{2-51}$$

The final result of this process is shown in Fig. 15, for a rectangular grid (examples of sheared grids can be found in Refs. 4 and 15). Note that the grid line in Fig. 15 which goes to downstream infinity does not originate at the trailing edge.

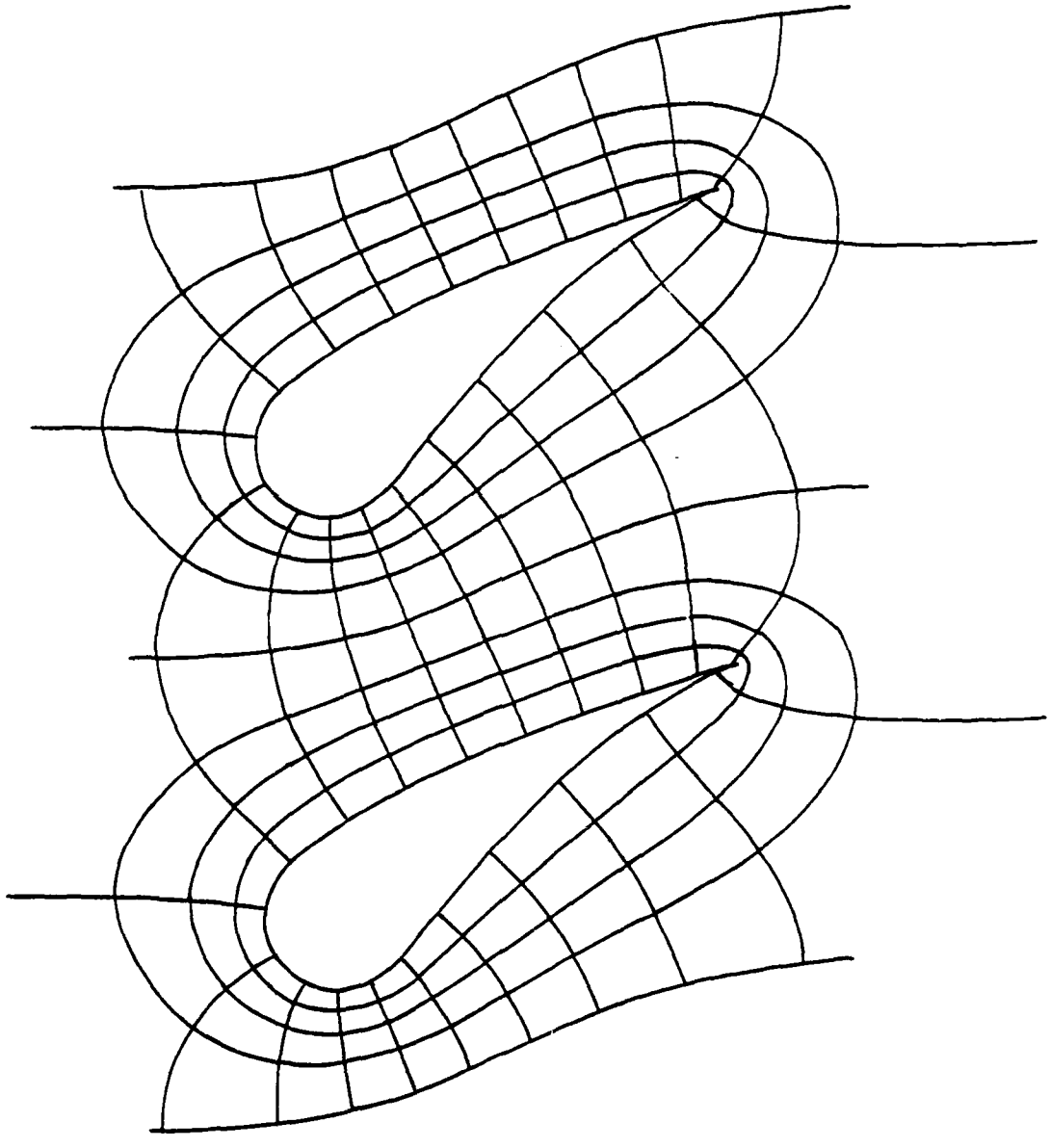


Figure 15. Coordinate Mapping

Section 3
METRIC EVALUATIONS

The coordinate transformation enters the flowfield solution algorithm only in the metric derivatives. These can be evaluated by differencing the coordinates themselves, or in the case of a conformal transformation, by evaluating the analytic expressions for them. These analytic expressions are derived in Ref. 1, and the code listed in that report contains the Fortran statements required to evaluate these expressions. However, as pointed out in Ref. 14, the truncation error resulting from the use of analytic metrics in the finite-difference flowfield code is large enough to cause major instabilities in the solution algorithm. It is highly preferable to use metrics that are found by differencing the coordinates in the same manner as the flowfield variables are differenced. A program to achieve this, for the grid conventions used in Ref. 14, is given in Appendix D.

Section 4
COMPUTER PROGRAM

A listing of the computer program is given in Appendix B, and a dictionary of variables is given in Appendix C. This section contains a general description of the program, plus some specific details.

In order to handle complex arithmetric, all variables beginning with the letter Z are declared to be complex by an implicit type specification at the beginning of the program.

The input is generally described by comment cards in the deck. Certain blade-shape parameters must be read in: EX, G, H, ZLE, ZTE, ZN, ZT. Also, if IGOT=1, ZC must be read in. The blade shape itself is defined by pairs of coordinates in the ξ , η plane - KJS on the suction side and KJP on the pressure side. In the version shown here, these were read in as a table in subroutine SHAPE.

The blade surface coordinates are numbered from 1 to KJMX; point number 1 is the trailing edge, 2 through KJLM are on the pressure side from trailing edge to leading edge, point KJLE is the leading-edge point (ZLE), KJLP through KJMXM are on the suction side from leading edge to trailing edge, and point KJMX repeats the trailing edge.

The complex sine functions are calculated next; then the iterations to determine the parameter C are done. The initial guess provided for C is $ZC = 1+i$. It may happen in some cases that a better guess is required: in particular, it is necessary that the value of C must lie outside the blade-surface curve in the Ω -plane. If it does not, then the interior of this curve in the Ω -plane is mapped to the exterior of the blade-surface image in the ω -plane. This fact can be seen from the discussion by Kober (Ref. 16) of the bilinear transformation applied to circles.

16. Kober, H., Dictionary of Conformal Transformations, Dover Publications, New York (1957).

After the parameter C has been found, the transformation to the ζ plane is carried out, using the fast Fourier transfrorm procedure (see Appendix A). The actual calculations of the Fourier coefficients are done in subroutine FFT2, a proprietary program of International Mathematical and Statistical Libraries, Inc. (IMSL). This routine computes the fast Fourier transform of a complex vector of length equal to a power of two (here 2^6). The coefficients of the input vector are given in normal order by the array named as the first argument of the call; the coefficients of the output vector are overstored in this array, in reverse binary order. The subroutine SHUFL is then used to restore this output to the normal order. The coefficients in the series expression for ζ are determined iteratively in a relaxation process that is terminated when the maximum change in θ falls below the tolerance ANGERR, or when IMX iterations are done.

In doing these iterations, it is necessary to know values of $\ln r$ at given values of θ; these are found by a spline fit in subroutine CISPLN, which is a straightforward implementation of the formulas given by Ahlberg, et al.[17]

In certain cases (typically when RMAX/RMIN is large) the calculated variation of θ with ϕ may be non-monotonic; if this occurs, the calculation should be repeated, with a smaller relaxation factor. The progress of the ϕ/θ iterations is printed, showing at each iteration the largest change in θ and the number of reversals (i.e., the number of occurrences of non-monotonic variation).

Next, the parameters ζ_A and ζ_B are found, starting with "best-guess" values calculated in the sectors described in Section 2. Once these are found, the mapping to the η -plane follows. The calculations that link the Ω -plane and the ζ -plane are done in subroutine OMETA, which sums the Theodorsen-Garrick series, using complex arithmetric. This subroutine has been modified slightly from that appearing in Ref. 1, as follows: the previous code evaluated sums of the form

17. Ahlberg, J.H., Nilson, E.N., and Walsh, J.L., The Theory of Splines and Their Applications Academic Press, New York (1967).

$$Z\,SUM = \sum_{JP=1}^{65} ZCC(JP)\ \zeta^{JP-1} \tag{4-1}$$

by a sequence of multiplications and additions:

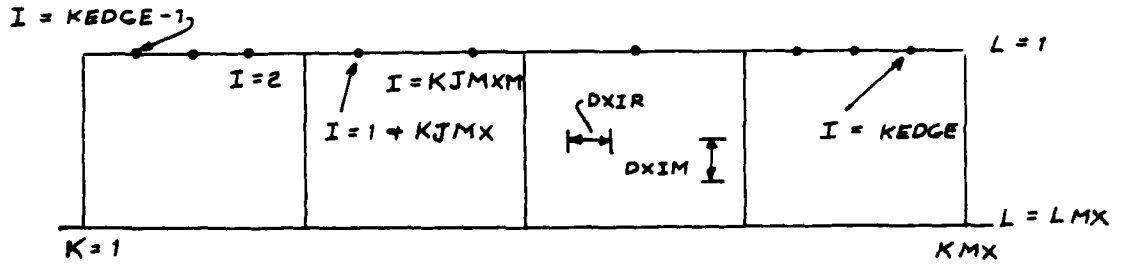
$$Z\,SUM = ZCC(65)\ \zeta^{64} + ZCC(64)\ \zeta^{63} + \ldots + ZCC(1) \tag{4-2}$$

The current version uses (Ref. 18, p. 28)

$$Z\,SUM = \left\{\left\{\left[ZCC(65)\zeta + ZCC(64)\right]\ \zeta + ZCC(63)\right\}\zeta + \ldots \right. \tag{4-3}$$

Finally, the blade-surface image is mapped into the $\hat{\xi}$ plane, using the elliptic-function formulas of Nielsen and Perkins[10] and of Luke,[12] as outlined in Section 2. This completes the mapping of the blade surface.

It is now possible to set up a grid in the $\hat{\xi}$ - plane, and map it back into the physical plane. This involves straightforward evaluations of the transformation functions. The only new complications are the need to evaulate the Jacobian elliptic sine (done in subroutine JCELFN) and to provide a good initial guess for the Newton-Raphson iteration used in finding $Z(\Omega)$. This guess is provided by starting each series of calculations on the image of the blade surface in the $\hat{\xi}$ plane, and interpolating to find the corresponding point in the S, n plane. The interpolation is done as follows: the blade-surface image values of $\hat{\xi}$ are stored in the array ZA1(I), where I ranges from 1 to KJMX. Each of these maps into a point in the S, n plane, which is stored in the array ZA2 (KJ), where KJ also ranges from 1 to KJMX. Thus the $\hat{\xi}$ plane has the appearance:



18. Hartree, D.R., Numerical Analysis, 2nd Edition, Oxford, Clarendon Press (1958).

Points in the uniform grid are denoted by K, L, where

$$Re(\hat{\xi}) = -3K(k) + (K-1)\,DXIR \qquad K = 1,\ KMX$$
$$Im(\hat{\xi}) = \frac{K'(k)}{2} - (L-1)\,DXIM \qquad L = 1,\ LMX \tag{4-4}$$

The value of I for which the real part of ZA1(I) is largest is denoted as KEDGE. Then, for a chosen value of K (with L = 1) the array ZA1(I) is searched (first for I = KEDGE to KJMX, then for I = 2 to KEDGE-1) until a value, called I^*, is found such that $Re[ZA1(I^*) < Re(\hat{\xi})]$. If none is found, it is concluded that I^* = KEDGE. Linear interpolation is then used,* between the points I^* and $I^* - 1$, to provide the initial guess. Fifty iterations are allowed for this step, at each value of K and L. If no solution is found, the value (0,0) is printed.

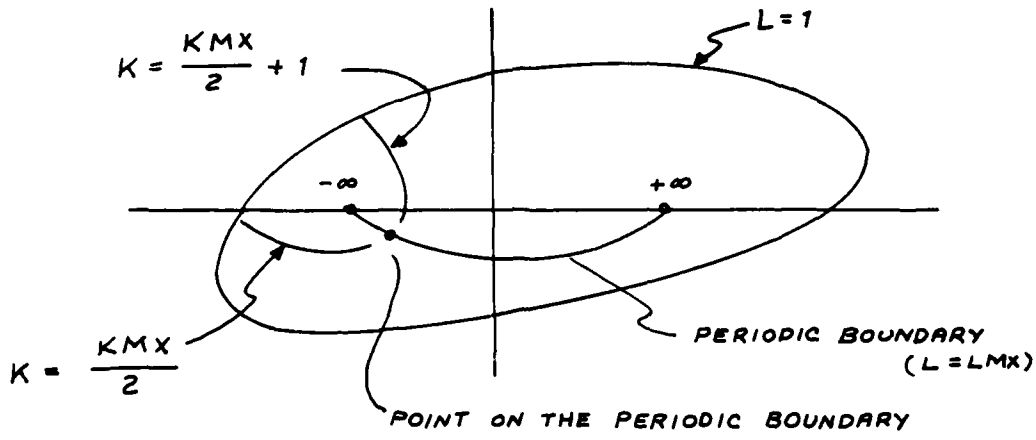
The calculation of the images of the grid points in the various planes is bypassed for the points at upstream and downstream infinity, and at the trailing edge. (The trailing-edge point will be a grid point if ISHEAR=1). The z -plane locations of upstream and downstream infinity are arbitrarily assigned to finite locations given by linear extrapolation from the two adjacent L -values.

The point at upstream infinity will be a grid point only if KMX is odd; in this case IOE=1, and the image calculations are bypassed.

Adjustments to the grid-point image locations in the z -plane are sometimes required for points on the periodic ouundary (L=LMX) for values of K near 1, KMX/2, and KMX. At these points, the Newton-Raphson iterations used in going from the Ω - plane to the z -plane sometimes cannot distinguish between points that are separated by $H + iG$. The problem can be seen best in the ω -plane. (The sketch below is for an even value of KMX; the same picture applies for an odd value):

---

*In Ref. 1, a quadratic interpolation was used. This introduced many complications, in order to avoid conditions where the interpolation base points straddle a sharp leading or trailing edge, and where I^* is near KEDGE or KEDGE-1. The logic in Ref. 1 was not adequate to handle all such cases.



Note that the same point in the ω -plane (and thus also in the Ω -plane) can map into either of two points in the Z -plane, which differ by $H + iG$. The selection is guided toward the correct value by starting on the blade (L=1) and working toward the periodic boundary (L=LMX), but it can happen that the wrong branch is chosen during the iterations. To avoid this, the imaginary parts of Z and ZN are compared, for K values near the leading edge, and the imaginary parts of Z and ZT are compared near the trailing edge, and the quantity $i \cdot SG$ is added or subtracted (depending on the value of K) where necessary.

Finally, the real and imaginary parts of Z are punched on cards (if PNCHZA=TRUE). These values can be used, in a separate program, to calculate the metrics of the transformation (see Appendix D).

Section 5
CONCLUDING REMARKS

The program described above has been applied to very few cases - the thick blades used for illustrative purposes in this report, and the cases shown in Refs. 4 and 15. Because of this limited experience, the range of applicability of the program is largely unknown. On the basis of current experience, it appears that the Theodorsen-Garrick step may not converge for gap/chord ratios less than around 0.8. The set of numerical tolerances, maximum iteration counts, and relaxation factors used may need to be adjusted for certain shapes. In addition, new approximations may be required as initial guesses in various places, in order to handle such features as leading edges with very small radii of curvature.

Appendix A

DETAILS OF THE FAST FOURIER TRANSFORM PROCEDURES

Equations 2-20 and 2-21 contain $2N+2$ constants, which are evaluated as follows: first, they are satisfied at a discrete number of points, denoted by ϕ_K :

$$\phi_K = \frac{2\pi(K-1)}{2N} \quad , \quad K = 1, 2, \ldots, 2N \tag{A-1}$$

where N was chosen to be 64, in the present case. Thus:

$$\ln r_K = A_0 + \sum_{j=1}^{N-1} (A_j \cos j\phi_K - B_j \sin j\phi_K) + (-1)^K A_N \tag{A-2}$$

$$\theta_K - \phi_K = B_0 + \sum_{j=1}^{N-1} (B_j \cos j\phi_K + A_j \sin j\phi_K) + (-1)^K B_N \tag{A-3}$$

Each right-hand side now contains $2N$ coefficients. The correspondence between either of these and the Fast Fourier Transform (FFT) as presented in Ref. 6 is given by the next two equations: consider the expression

$$Y(q) = \sum_{n=0}^{2N-1} C(n) W_{2N}^{nj} \quad ; \quad j = 0, 1, 2, \ldots, 2N-1 \; ; \; W_{2N} = e^{i\frac{2\pi}{2N}} \tag{A-4}$$

where values $Y(\cdot)$ are real, and the $2N$ values of $C(\cdot)$ are in general complex, but must satisfy the following redundancy condition, in order that the $Y(\cdot)$ values be real:

$$C(n) = \tilde{C}(2N-n) \quad , \quad n = 1, 2, \ldots, N-1 \tag{A-5}$$

$C(0)$ and $C(N)$ pure real

where the tilde denotes the complex conjugate. When these conditions are met, Eq. A-4 can be written as

$$Y(l) = C_R(0) + (-1)^l C_R(N) + 2\sum_{n=1}^{N-1} \left\{ C_R(n) \cos\frac{nl\pi}{N} - C_I(n) \sin\frac{nl\pi}{N} \right\}$$
$$l = 0, 1, \ldots, 2N-1 \tag{A-6}$$

This form can now be used, in conjunction with Eq. 2-20 or 2-21, to facilitate application of the FFT to the complex form given in Eq. A-4.

In the case of Eq. A-2, values of the coefficients A_0 through A_N and B_1 through B_{N-1} are found, from given values of $\ln r$. This is procedure 4 of Ref. 6, which takes the following steps:

1. Set
$$\left.\begin{aligned} X_1(k) &= Y(2k) = (\ln r)_{2k} \\ X_2(k) &= Y(2k+1) = (\ln r)_{2k+1} \end{aligned}\right\} \quad k = 0, 1, \ldots, N-1 \qquad \text{(A-7)}$$

2. Set
$$X(j) = X_1(j) + i X_2(j) \quad , \quad j = 0, 1, \ldots, N-1 \qquad \text{(A-8)}$$

3. Calculate the N-point Discrete Fourier Transform of

$$A(n) = A_1(n) + iA_2(n) = \frac{1}{N} \sum_{j=0}^{N-1} X(j)\, W_N^{-nj} \; ; \quad n = 0, 1, \ldots, N-1 \qquad \text{(A-9)}$$

$$W_N = e^{i\frac{2\pi}{N}}$$

4. By periodicity, set $A(N) = A(0)$

5. Apply Eq. 34 of Ref. 6, in order to extract $A_1(n)$ and $A_2(n)$ from $A(n)$:

$$\left.\begin{aligned} A_1(n) &= \frac{1}{2}\left\{\tilde{A}(N-n) + A(n)\right\} \\ A_2(n) &= \frac{i}{2}\left\{\tilde{A}(N-1) - A(n)\right\} \end{aligned}\right\} \quad n = 0, 1, \ldots, \frac{N}{2} \qquad \text{(A-10)}$$

Note that these expressions use $A(n)$ for $n = 0, 1, .., N$ to give $A_1(n)$ and $A_2(n)$ for $n = 0, 1, \ldots, N/2$

6. These values of $A_1(n)$ and $A_2(n)$ then give $C(n)$ for the same range:

$$C(n) = \frac{1}{2}\left[A_1(n) + W_{2N}^{-n} A_2(n)\right], \quad n = 0, 1, \ldots, N/2 \qquad \text{(A-11)}$$

For the range of n from $\frac{N}{2}+1$ to $N-1$, use Eq. 36 of Ref. 6, with n replaced by $N-n$ (and noting that $W_{2N}^{-N} = -1$):

$$C(n) = \tilde{C}(2N-n) \qquad \text{(A-12)}$$

$$= \frac{1}{2}\left\{A_1(N-n) + W_{2N}^{n} A_2(N-n)\right\} \quad n = \frac{N}{2}+1, \frac{N}{2}+2, \ldots, N-1$$

This equation, applied for $n = \frac{N}{2}+1, \frac{N}{2}+2, \ldots, N-1$ uses A_1 and A_2 with index $\frac{N}{2}-1, \frac{N}{2}-2, \ldots, 1$ to get $C(n)$ for $n = \frac{N}{2}+1, \frac{N}{2}+2, \ldots, N-1$. The process is completed by setting

$$C(N) = \frac{1}{2}\left\{A_1(0) - A_2(0)\right\} \tag{A-13}$$

7. The A_j and B_j coefficients are retrieved from:

$$A_0 = Re\left[C(0)\right], \quad A_N = Re\left[C(N)\right]$$
$$A_j = 2\,Re\left[C(j)\right], \quad B_j = 2\,Im\left[C(j)\right], \quad j = 1, 2, \ldots, N-1 \tag{A-14}$$

At this point, B_0 and B_N are undetermined. Following Ives, B_N is set equal to zero, and B_0 is chosen so as to place the trailing edge at $\phi = 0$ (this latter selection of B_0 is actually carried out in a subsequent step, noted below).

In the case of Eq. A-3, the A's and B's are considered known, and are used to evaluate θ_k. The coefficient B_0 can be found from

$$\theta_{TE} = B_0 + \sum_{j=1}^{N-1} B_j \quad (B_N = 0) \tag{A-15}$$

Actually, it is simpler to evaluate the right-hand side of

$$\theta_k - \phi_k - B_0 = \sum_{j=1}^{N-1} (B_j \cos j\phi_k + A_j \sin j\phi_k), \quad k = 0, 1, \ldots, N-1 \tag{A-16}$$

and then find B_0 from

$$B_0 = \theta_0 - RHS)_{k=0} \tag{A-17}$$

The actual evaluation of the right-hand side takes the following steps (Procedure 5 of Ref. 6): by comparison of Eqs. A-3 and A-6:

1. Set $C(0) = 0, \quad C(N) = 0$

$$C(n) = \frac{1}{2}\left[B_n - i\,A_n\right], \quad n = 1, 2, \ldots, N-1 \tag{A-18}$$

Note that the C's determined here are different from those used in Procedure 4; the A_j and B_j values are the same, but their relation to C_j is different.

2. Values of C_j are then used to find $A_1(n)$ and $A_2(n)$: Equations 40 and 41 of Ref. 6 are rewritten, using

$$C(n) = \tilde{C}(2N-n)$$

Replace n by $N+n$:

$$C(N+n) = \tilde{C}(2N-N-n) = \tilde{C}(N-n)$$

Thus Eqs. 40 and 41 of Ref. 6 are

$$\left.\begin{aligned} A_1(n) &= C(n) + \tilde{C}(N-n) \\ A_2(n) &= \left[C(n) - \tilde{C}(N-n)\right] W_{2N}^{n} \end{aligned}\right\} \quad n = 0, 1, \ldots, N/2 \qquad \text{(A-19)}$$

These are all the values needed for A_1 and A_2.

3. Find $A(n)$, $n=0,1,\ldots,N-1$ from Eqs. 42 and 43 of Ref. 6:

$$\left.\begin{aligned} A(n) &= A_1(n) + i A_2(n) \\ A(N-n) &= \tilde{A}_1(n) + i \tilde{A}_2(n) \end{aligned}\right\} \quad n = 0, 1, \ldots, N/2 \qquad \text{(A-20)}$$

This gives, on the left-hand side, all values from $n=0$ to $n=N$.

4. Calculate

$$X(j) = \sum_{n=0}^{N-1} A(n) W_N^{nj}, \quad j = 0, 1, \ldots, N-1 \qquad \text{(A-21)}$$

5. Finally:

$$\left.\begin{aligned} \theta_{2k} - \phi_{2k} - B_0 &= Re\left[X(k)\right] \\ \theta_{2k+1} - \phi_{2k+1} - B_0 &= Im\left[X(k)\right] \end{aligned}\right\} \quad k = 0, 1, \ldots, N-1 \qquad \text{(A-22)}$$

The first of these equations, with $k=0$, is used to find B_0.

In order to apply these formulas, it is necessary to have a relation

between r and θ:

$$r_K = r(\theta_K) \quad , \quad K = 1, 2, \ldots, 2N \tag{A-23}$$

which is found from a spline fit to the blade-surface image in the ω- plane.

The discrete Fourier transform and its inverse are given by

$$A(n) = \frac{1}{N} \sum_{j=0}^{N-1} X(j) \, W_N^{-nj} \quad , \quad n = 0, 1, 2, \ldots, N-1 \tag{A-24}$$

$$X(j) = \sum_{n=0}^{N-1} A(n) \, W_N^{nj} \quad , \quad j = 0, 1, 2, \ldots, N-1 \tag{A-25}$$

The IMSL routine FFT2 evaluates the second of these, i.e., it returns $X(j)$, given the values $A(n)$. To evaluate the first of these, FFT2 is used with input $\tilde{X}(j) / N$, and with output interpreted to be $\tilde{A}(n)$; this is Procedure 1 of Reference 6:

$$N \tilde{A}(n) = \sum_{j=0}^{N-1} \tilde{X}(j) \, W_N^{nj} \tag{A-26}$$

In the FORTRAN version of these and other procedures, it is convenient to use indices that begin at one, rather than zero, by setting $j+1 = j'$ (FORTRAN symbol JP). The corresponding table, for example of the coefficient B_j, is

j	JP	B_j	B(JP)
0	1	B_o	B(1)
1	2	B_1	B(2)
2	3	B_2	B(3)
N-1	N	B_{N-1}	B(N)
N	N+1	B_N	B(N+1)

In addition, care must be taken with the argument $N-n$; for example, Eqs. are written as

$$\left.\begin{aligned} \mathcal{Z}A1(NP) &= \mathcal{Z}CC(NP) + \widetilde{\mathcal{Z}CC}(N+2-NP) \\ \mathcal{Z}A2(NP) &= \left[\mathcal{Z}CC(NP) - \widetilde{\mathcal{Z}CC}(N+2-NP)\right] W_{2N}^{n} \end{aligned}\right\} \quad NP = 1,2,\ldots,\frac{N}{2}+1 \tag{A-27}$$

It can be verified that the quantity N + 2 - NP in the arguments above preserves the correct ordering - for example when N = 64, and n = 0, $\tilde{C}(N-n) = \tilde{C}(64)$ This would be stored in ZCC(64 + 2 - 1) = ZCC(65).

Appendix B

COMPUTER PROGRAM LISTING

```
LEVEL 21.7 ( DEC 72 )                          OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
                C
                C  PROGRAM IVLTMP - THE IVES - LIUTERMOZA CONFORMAL TRANSFORMATION FOR
                C  TURBOMACHINERY CASCADES (AIAA JOURNAL,VOL. 5,1977, PP 647 - 652)
                C  DOCUMENTATION IS GIVEN IN:
                C       W. J. RAE, A COMPUTER PROGRAM FOR THE IVES TRANSFORMATION
                C       IN TURBOMACHINERY CASCADES, CALSPAN CORPORATION REPORT 6275-A-3,
                C       NOVEMBER 1980
                C       W.J. RAE, MODIFICATIONS OF THE IVES - LIUTERMOZA CONFORMAL-
                C       MAPPING PROCEDURE FOR TURBOMACHINERY CASCADES, ASME PAPER
                C       83-GT-116, MARCH 1983
                C       W.J. RAE, REVISED COMPUTER PROGRAM FOR EVALUATING THE IVES
                C       TRANSFORMATION IN TURBOMACHINERY CASCADES, CALSPAN CORPORATION
                C       REPORT 7177-A-1, JULY 1983
                C
  ISN 0002            IMPLICIT REAL*8(A-H,O-Y),COMPLEX*16(Z)
  ISN 0003            LOGICAL PNCHZA
  ISN 0004            COMMON/TGINTG/N,NP1,NP2,N2,NB2,NB2P1,IP,IPMX,ITP,IWK,IMX,KJMX
  ISN 0005            COMMON/TGCMPX/Z1,ZW2N,ZI,ZNN,ZA,ZCC,ZA1,ZA2
  ISN 0006            COMMON/TGDBLE/PBN,OM,OMM,ANGERR,A,B,E,F,THT,PHI,X,Y
  ISN 0007            DIMENSION ZS(80),ZP(80),ZOMS(80),ZOMP(80)
  ISN 0008            DIMENSION RDS(80),RDP(80),THS(80),THP(80)
  ISN 0009            DIMENSION RDSX(80),RDPX(80),THSX(80),THPX(80)
  ISN 0010            DIMENSION X(160),Y(160),E(150),F(150),THT(160)
  ISN 0011            DIMENSION PHI(130),A(65),B(65),ZCC(65),
                     *    ZA(165),ZA1(165),ZA2(165),
                     *            IWK(7)
  ISN 0012            DIMENSION ITP(100),ID(36)
  ISN 0013            DIMENSION XX(50,20),YY(50,20)
  ISN 0014            DO 10 I = 1,7
  ISN 0015         10 IWK(I) = 0
                C
  ISN 0016            NAMELIST/INPUTS/ ANGERR,EX,G,H,IGOT,ILE,ITE,KMX,
                     *            IMX, LMX,  OM, PNCHZA, RTOL, ZC, ZLE, ZN, ZT, ZTE,
                     *            IPMX,ISHEAR,INR,KJS,KJP,KNR
                C
                C --- SET NAMELIST DEFAULT VALUES
                C
  ISN 0017            ANGERR=0.01D0
  ISN 0018            IGOT=0
  ISN 0019            ILE = 1
  ISN 0020            ITE=0
  ISN 0021            KMX = 40
  ISN 0022            LMX = 10
  ISN 0023            KJS = 20
  ISN 0024            KJP = 20
  ISN 0025            IMX = 400
  ISN 0026            ISHEAR = 1
  ISN 0027            IPMX = 1
  ISN 0028            KNR = 0
  ISN 0029            INR = 0
  ISN 0030            OM=0.1D0
  ISN 0031            PNCHZA=.FALSE.
  ISN 0032            RTOL=3.0D0
                C
                C     NOTE: EX, G, H, ZC, ZLE, ZN, ZT, ZTE HAVE NO DEFAULT VALUES
                C     (ZC IS NOT NEEDED IF IGOT=0)
```

```
                C
ISN 0033            READ(5,103)(ID(I),I=1,36)
ISN 0034          103 FORMAT(18A4)
                C
ISN 0035            PI = 4.0D0*DATAN(1.0D0)
ISN 0036            TPI = 2.0D0*PI
ISN 0037            ZPI = DCMPLX(PI,0.0D0)
ISN 0038            Z1 = DCMPLX(1.0D0,0.0D0)
ISN 0039            ZERO = DCMPLX(0.0D0,0.0D0)
ISN 0040            ZMGA = DCMPLX(+1.0D0,0.0D0)
ISN 0041            ZMGB = DCMPLX(-1.0D0,0.0D0)
                C *** READ PNCHZA=T IF (ZA(L),L=1,LMX) IS TO BE PUNCHED FOR ALL VALUES
                C *** OF K, OTHERWISE READ PNCHZA=F
                C  IGOT = 1 IF THE VALUE OF ZC IS KNOWN.  OTHERWISE, IGOT = 0.
                C  KMX  AND LMX  ARE THE GRID SIZES IN THE FINAL TRANSFORMED PLANE.
                C  KJS AND KJP ARE THE NUMBERS OF POINTS ON THE SUCTION AND PRESSURE
                C  SIDES AT WHICH PAIRS OF BLADE COORDINATES WILL BE INPUT.
                C  ILE = 0 OR 1 FOR A SHARP OR ROUNDED LEADING EDGE, RESPECTIVELY
                C  ITE = 0 OR 1 FOR A SHARP OR ROUNDED TRAILING EDGE, RESPECTIVELY.
                C  IMX IS THE MAXIMUM NUMBER OF ITERATIONS ALLOWED FOR THE PHI/THETA
                C  ITERATIONS
                C  INR AND KNR.NE.0 WILL CAUSE A DIAGNOSTIC OUTPUT FOR THE FIRST INR
                C  NEWTON-RAPHSON ITERATIONS (TO FIND Z, GIVEN ZBOMK), AT THE STATION
                C  K = KNR.  THE PROGRAM WILL THEN STOP.
                C  IPMX.NE.1    CAN BE USED TO DISPLAY THE VALUES OF THETA DURING THE
                C  PHI/THETA ITERATIONS, AT THE ITERATION NUMBERS READ INTO THE
                C  ITP ARRAY BELOW.
                C  ISHEAR = 0 GIVES AN ORTHOGONAL GRID.  THE LINES K=1 AND K=KMX, WHICH
                C  START AT THE TRAILING EDGE, DO NOT GO TO DOWNSTREAM INFINITY.
                C  ISHEAR = 1 SHEARS THE GRID UNIFORMLY; IN THIS CASE, THE K=1 AND KMX
                C  LINES DO GO TO DOWNSTREAM INFINITY.
                C  OM IS A RELAXATION FACTOR USED IN THE PHI/THETA MAPPING.  USE 0.1,
                C  OR A SMALLER VALUE IF THE A AND B ITERATIONS FAIL TO CONVERGE.
                C  ANGERR IS THE ANGULAR TOLERANCE (IN RADIANS) FOR THE PHI/THETA
                C  TRANSFORMATION.  A REASONABLE VALUE IS 0.01
                C  RTOL IS THE TOLERANCE FOR THE MAX/MIN RADIUS RATIO IN THE
                C  OMEGA PLANE
                C
                C --- FOR A SHARP LEADING EDGE (ILE=0) ZN MUST EQUAL ZLE
                C --- FOR A SHARP TRAILING EDGE (ITE=0) ZT MUST EQUAL ZTE
                C
                C --- READ NAMELIST INPUT DATA
                C
ISN 0042            READ(5,INPUTS)
ISN 0043            ITP(1) = 999
ISN 0044            IF(IPMX.NE.1) READ(5,102)(ITP(IP),IP=1,IPMX)
ISN 0046          102 FORMAT(20I4)
ISN 0047            IP = 1
                C
                C
ISN 0048            KMXH=(KMX+1)/2
ISN 0049            KMXL=KMXH
ISN 0050            IF(MOD(KMX,2).NE.0) KMXL=KMXH-1
ISN 0052            IOE = MOD(KMX,2)
ISN 0053            KAA = KMXH - 2 - IOE
ISN 0054            KBB = KMXH + 3
ISN 0055            KMXM4 = KMX - 4
ISN 0056            KOUNT = 0
```

```
C
C
C  INPUT VARIABLES EX, G, H, ZLE, ZN, ZT, ZTE,
C  THE FORTRAN STATEMENTS IN SUBROUTINE SHAPE,
C  AND THE VARIABLES LISTED IN COMMON BLOCK GEOM (IF ONE IS BEING USED)
C  ARE ALL SPECIFIC TO THE BLADE SHAPE BEING USED.
C
C
C  CALCULATION OF THE BLADE SHAPE
C
ISN 0057        D = CDABS(ZTE-ZLE)
ISN 0058        CALL SHAPE(D,H,G,EX,ZP,ZS,KJS,KJP)
ISN 0059        SG = DSQRT(H*H+G*G)
ISN 0060        SIZE = 5.0D0*SG
ISN 0061        ZDA = DCMPLX(G/SG,-H/SG)
ISN 0062        ZGAMMA=DCONJG(ZDA)
ISN 0063        WRITE(6,207)
ISN 0064      207 FORMAT(1H1)
ISN 0065        WRITE(6,206)(ID(I),I=1,36)
ISN 0066      206 FORMAT(30X,18A4)
ISN 0067        WRITE(6,240) D,H,G,EX,SG
ISN 0068      240 FORMAT(//10X,'BLADE-GEOMETRY PARAMETERS ARE:',
     *    //5X,'ABS(ZTE-ZLE) =',
     *              F10.5,'  H = ',F10.5,'  G = ',F10.5,'  EX = ',
     *    F10.5,'  SLANT GAP =',F10.5,//)
ISN 0069        WRITE(6,INPUTS)
ISN 0070        KJLE = KJP + 2
ISN 0071        KJMX = KJLE + KJS + 1
ISN 0072        WRITE(6,209)
ISN 0073      209 FORMAT( 8X,'BLADE COORDINATES:',
     *    //9X,'SUCTION SIDE',/3X,'KJ',7X,'S',13X,'N',13X,'X',13X,'Y',/)
ISN 0074        ZXY = ZGAMMA*ZLE
ISN 0075        WRITE(6,270) KJLE,ZLE,ZXY
ISN 0076      270 FORMAT(I5,1P4E14.5)
ISN 0077        DO 64 K = 1,KJS
ISN 0078        KJ = KJLE + K
ISN 0079        ZXY = ZGAMMA*ZS(K)
ISN 0080        WRITE(6,270) KJ,ZS(K),ZXY
ISN 0081       64 CONTINUE
ISN 0082        ZXY = ZGAMMA*ZTE
ISN 0083        WRITE(6,270) KJMX,ZTE ,ZXY
ISN 0084        WRITE(6,271)
ISN 0085      271 FORMAT(//8X,'PRESSURE SIDE',/3X,'KJ',7X,'S',13X,'N',
     *    13X,'X',13X,'Y',/)
ISN 0086        ZXY = ZGAMMA*ZLE
ISN 0087        WRITE(6,270) KJLE,ZLE,ZXY
ISN 0088        DO 61 K = 1,KJP
ISN 0089        KJ = KJLE - K
ISN 0090        ZXY = ZGAMMA*ZP(K)
ISN 0091        WRITE(6,270) KJ,ZP(K),ZXY
ISN 0092       61 CONTINUE
ISN 0093        KJ = 1
ISN 0094        ZXY = ZGAMMA*ZTE
ISN 0095        WRITE(6,270) KJ,ZTE,ZXY
C
ISN 0096        ZDN = DCMPLX(H,G)
ISN 0097        ZZ = (ZT-ZN)/ZDN
ISN 0098        ZTN = ZPI*ZZ
```

```
ISN 0099            CHI = PI*EX*(G*DREAL(ZT-ZN)-H*DIMAG(ZT-ZN))/(SG*SG)
ISN 0100            XA = PI*EX*(H*DREAL(ZT-ZN)+G*DIMAG(ZT-ZN))/(SG*SG)
ISN 0101            R = DEXP(-CHI)
ISN 0102            ZPLUS = DCMPLX(R*DCOS(XA),-R*DSIN(XA))
ISN 0103            R = 1.0D0/R
ISN 0104            ZMINUS= DCMPLX(R*DCOS(XA),R*DSIN(XA))
ISN 0105            DO 20 K = 1,KJS
ISN 0106            ZZT = (ZS(K)-ZT)/ZDN
ISN 0107            ZETA1S = ZPI*ZZT
ISN 0108            ZETA2S = CDSIN(ZETA1S)
ISN 0109            ZZN = (ZS(K)-ZN)/ZDN
ISN 0110            ZETA3S = ZPI*ZZN
ISN 0111            ZETA4S = CDSIN(ZETA3S)
ISN 0112            ZFS = ZETA2S/ZETA4S
ISN 0113            RDS(K) = CDABS(ZFS)
ISN 0114            THS(K)= DATAN2(DIMAG(ZFS),DREAL(ZFS))
ISN 0115         20 CONTINUE
ISN 0116            DO 24 K = 1,KJP
ISN 0117            ZZT = (ZP(K)-ZT)/ZDN
ISN 0118            ZETA1P = ZPI*ZZT
ISN 0119            ZETA2P = CDSIN(ZETA1P)
ISN 0120            ZZN = (ZP(K)-ZN)/ZDN
ISN 0121            ZETA3P = ZPI*ZZN
ISN 0122            ZETA4P = CDSIN(ZETA3P)
ISN 0123            ZFP = ZETA2P/ZETA4P
ISN 0124            RDP(K) = CDABS(ZFP)
ISN 0125            THP(K)= DATAN2(DIMAG(ZFP),DREAL(ZFP))
ISN 0126         24 CONTINUE
                 C  NOW ADD THE LEADING- AND TRAILING-EDGE POINTS, AND STORE THE G(Z)
                 C  ARRAY AS E(KJ)*EXP(I*THT(KJ)),WHERE KJ=1,KJMX AS YOU GO FROM TE AROUND
                 C  THE PRESSURE SIDE TO THE LE (KJ=KJLE) AND THEN ALONG THE SUCTION SIDE
                 C  BACK TO THE TE AGAIN (KJ=KJMX).
ISN 0127            IF(ITE.EQ.1) GO TO 13
ISN 0129            E(1)=0.0D0
ISN 0130            THT(1)=0.0D0
ISN 0131            GO TO 14
ISN 0132         13 ZETA2=CDSIN(ZPI*(ZTE-ZT)/ZDN)
ISN 0133            ZETA4=CDSIN(ZPI*(ZTE-ZN)/ZDN)
ISN 0134            ZFP=ZETA2/ZETA4
ISN 0135            E(1)=CDABS(ZFP)
ISN 0136            THT(1)=DATAN2(DIMAG(ZFP),DREAL(ZFP))
ISN 0137         14 IF(ILE.EQ.1) GO TO 15
ISN 0139            E(KJLE) = 0.0D0
ISN 0140            THT(KJLE) = 0.0D0
ISN 0141            GO TO 16
ISN 0142         15 ZETA2=CDSIN(ZPI*(ZLE-ZT)/ZDN)
ISN 0143            ZETA4=CDSIN(ZPI*(ZLE-ZN)/ZDN)
ISN 0144            ZFP=ZETA2/ZETA4
ISN 0145            E(KJLE) = CDABS(ZFP)
ISN 0146            THT(KJLE) = DATAN2(DIMAG(ZFP),DREAL(ZFP))
ISN 0147         16 DO 17 K = 1,KJP
ISN 0148            KJ = KJLE - K
ISN 0149            E(KJ)=RDP(K)
ISN 0150            THT(KJ)=THP(K)
ISN 0151         17 CONTINUE
ISN 0152            DO 18 K = 1,KJS
ISN 0153            KJ = KJLE + K
ISN 0154            E(KJ)=RDS(K)
```

```
ISN 0155          13 THT(KJ)=THS(K)
ISN 0156             E(KJMX) = E(1)
ISN 0157             THT(KJMX) = THT(1)
                C
                C  NOW ADJUST THE BRANCHES OF G(Z) SO AS TO BE CONTINUOUS ACROSS
                C  THE CUT    (ALONG THE NEGATIVE REAL AXIS) OF THE DATAN2 FUNCTION.
                C
ISN 0158             IF(H.LT.0.0D0) GO TO 251
ISN 0160             IF(ITE.EQ.1) GO TO 251
                C
                C  FOR COMPRESSORS WITH A SHARP T. E. , THE CONVENTION IS TO FORCE
                C  ARG(THT(KJ=2)) TO BE NEGATIVE:
                C
ISN 0162             BR = 0.0D0
ISN 0163             KA = 3
ISN 0164             PO = THT(2)
ISN 0165             IF(PO.LT.0.0D0) GO TO 252
ISN 0167             BR = -1.0D0
ISN 0168             THT(2) = THT(2) - TPI
ISN 0169             GO TO 252
                C
                C  CONVENTION FOR ALL TURBINES, AND FOR COMPRESSORS
                C  WITH A ROUND T. E., IS:
                C
ISN 0170        251  BR=0.0D0
ISN 0171             KA=3
ISN 0172             IF(ITE.EQ.1) KA=2
ISN 0174             PO=THT(KA-1)
ISN 0175        252  DO 321 KJ=KA,KJMX
ISN 0176             CHG=THT(KJ)-PO
ISN 0177             PO=THT(KJ)
ISN 0178             IF(DABS(CHG).LE.PI) GO TO 321
ISN 0180             IF(CHG.GT.PI) BR=BR-1.0D0
ISN 0182             IF(CHG.LT.-PI) BR=BR+1.0D0
ISN 0184        321  THT(KJ)=THT(KJ)+BR*TPI
ISN 0185             DO 323 K=1,KJP
ISN 0186             THP(K) = THT(KJLE-K)
ISN 0187        323  CONTINUE
ISN 0188             DO 322 K = 1,KJS
ISN 0189             THS(K) = THT(KJLE+K)
ISN 0190        322  CONTINUE
                C
ISN 0191             DO 25 K = 1,KJS
ISN 0192             ARS = EX*THS(K)
ISN 0193             RS = RDS(K)**EX
ISN 0194             RDSX(K) = RS
ISN 0195             THSX(K) = ARS
ISN 0196         25  CONTINUE
ISN 0197             DO 26 K = 1,KJP
ISN 0198             ARP = EX*THP(K)
ISN 0199             RP = RDP(K)**EX
ISN 0200             RDPX(K) = RP
ISN 0201             THPX(K) = ARP
ISN 0202         26  CONTINUE
ISN 0203             WRITE(6,241)
ISN 0204        241  FORMAT(//10X,'BLADE-SURFACE IMAGES IN THE G - PLANE (RATIO OF',
                    *    ' SINES) AND CAP OMEGA PLANE(G**1/KAPPA) ,AND',
                    *    // 9X,' RADII AND ANGLES USED IN SELECTING THE PROPER',
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                *    ' BRANCHES OF THE RATIO OF SINE FUNCTIONS ARE:',
                *    //3X,'KJ',15X,'G',25X,'CAP OMEGA',13X,'R',11X,'THETA',
                *    9X,'R**EX',7X,'THETA*EX' )
ISN 0205            XS=E( 1)*DCOS(THT( 1))
ISN 0206            YS=E( 1)*DSIN(THT( 1))
ISN 0207            RDTX = E(1)**EX
ISN 0208            THTX = EX*THT(1)
ISN 0209            US = RDTX*DCOS(THTX)
ISN 0210            VS = RDTX*DSIN(THTX)
ISN 0211            WRITE(6,325) XS,YS,US,VS,E(1),THT(1),RDTX,THTX
ISN 0212         232 FORMAT(I5,1P8E14.5)
ISN 0213         324 FORMAT('  LE ',1P8E14.5)
ISN 0214         325 FORMAT('  TE ',1P8E14.5)
ISN 0215            KJLM = KJLE - 1
ISN 0216            DO 65 KJ = 2,KJLM
ISN 0217            K = KJLE - KJ
ISN 0218            XP = RDP(K)*DCOS(THP(K))
ISN 0219            YP = RDP(K)*DSIN(THP(K))
ISN 0220            UP = RDPX(K)*DCOS(THPX(K))
ISN 0221            VP = RDPX(K)*DSIN(THPX(K))
ISN 0222            WRITE(6,232)KJ,XP,YP,UP,VP,RDP(K),THP(K),RDPX(K),THPX(K)
ISN 0223         65 CONTINUE
ISN 0224            XS = E(KJLE)*DCOS(THT(KJLE))
ISN 0225            YS = E(KJLE)*DSIN(THT(KJLE))
ISN 0226            RDLX = E(KJLE)**EX
ISN 0227            THLX = EX*THT(KJLE)
ISN 0228            US = RDLX*DCOS(THLX)
ISN 0229            VS = RDLX*DSIN(THLX)
ISN 0230            WRITE(6,324)XS,YS,US,VS,E(KJLE),THT(KJLE),RDLX,THLX
ISN 0231            KJMXM = KJMX - 1
ISN 0232            KJLP = KJLE + 1
ISN 0233            DO 31 KJ = KJLP,KJMXM
ISN 0234            K = KJ - KJLE
ISN 0235            XS = RDS(K)*DCOS(THS(K))
ISN 0236            YS = RDS(K)*DSIN(THS(K))
ISN 0237            US = RDSX(K)*DCOS(THSX(K))
ISN 0238            VS = RDSX(K)*DSIN(THSX(K))
ISN 0239            WRITE(6,232) KJ,XS,YS,US,VS,RDS(K),THS(K),RDSX(K),THSX(K)
ISN 0240         31 CONTINUE
ISN 0241            XP = E(KJMX)*DCOS(THT(KJMX))
ISN 0242            YP = E(KJMX)*DSIN(THT(KJMX))
ISN 0243            RDTX = E(KJMX)**EX
ISN 0244            THTX = EX*THT(KJMX)
ISN 0245            UP = RDTX*DCOS(THTX)
ISN 0246            VP = RDTX*DSIN(THTX)
ISN 0247            WRITE(6,325)XP,YP,UP,VP,E(KJMX),THT(KJMX),RDTX,THTX
ISN 0248            WRITE(6,242)ZPLUS,ZMINUS
ISN 0249         242 FORMAT(//10X,'POINTS AT INFINITY ARE LOCATED IN THE CAP OMEGA',
                *    ' PLANE AT:',
                *    //15X,'PLUS:',1P2E15.4,'   MINUS:',2E15.4,//)
                C
                C  DETERMINATION OF ZC SUCH AS TO MINIMIZE THE RATIO RMAX/RMIN IN THE
                C  L.C. OMEGA PLANE
                C
ISN 0250            M=1
ISN 0251            ZE = (ZMGA-ZMGB)/(ZPLUS-ZMINUS)
ISN 0252            ZF = (ZMGA*ZMINUS-ZMGB*ZPLUS)/(ZPLUS-ZMINUS)
ISN 0253            ZG = (ZMGA*ZPLUS-ZMGB*ZMINUS)/(ZPLUS-ZMINUS)
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ISN 0254            WRITE(6,601)
ISN 0255          601 FORMAT('1ITER',11X,'ZD',22X,'ZB',22X,'ZC',18X,'ZOMSTR',19X,'ZNTRD'
                     1/13X,'RMIN',8X,'RMAX',7X,'RATIO'/' (ZA(KJ),KJ=1,KJMX)')
ISN 0256          250 ITER=1
ISN 0257            RATIO=0.0D0
ISN 0258            IF(IGOT.EQ.1) GO TO 60
                  C
                  C  FOR A FIRST GUESS, USE ZC=(-1.0,+1.0)
                  C
ISN 0260            ZC=DCMPLX(-1.0D0,1.0D0)
                  C
ISN 0261           60 ZB = (ZMGA*ZPLUS-ZMGB*ZMINUS-ZC*(ZMGA-ZMGB))/(ZPLUS-ZMINUS)
ISN 0262            ZD = (ZMGB*ZPLUS-ZMGA*ZMINUS+(ZMGA-ZMGB)/ZC)/(ZPLUS-ZMINUS)
ISN 0263           68 CONTINUE
ISN 0264            DO 75 K = 1,KJS
ISN 0265            RS = RDSX(K)
ISN 0266            ARS = THSX(K)
ISN 0267            ZOMS(K) = DCMPLX(RS*DCOS(ARS),RS*DSIN(ARS))
ISN 0268            ZOMS(K) = (ZD-ZB*ZOMS(K)/ZC)/(Z1-ZOMS(K)/ZC)
ISN 0269           75 CONTINUE
ISN 0270            DO 85 K = 1,KJP
ISN 0271            RP = RDPX(K)
ISN 0272            ARP = THPX(K)
ISN 0273            ZOMP(K) = DCMPLX(RP*DCOS(ARP),RP*DSIN(ARP))
ISN 0274            ZOMP(K) = (ZD-ZB*ZOMP(K)/ZC)/(Z1-ZOMP(K)/ZC)
ISN 0275           85 CONTINUE
ISN 0276            IF(ILE.EQ.1) GO TO 11
ISN 0278            ZOMLE = ZB
ISN 0279            GO TO 12
ISN 0280           11 RD = E(KJLE)
ISN 0281            TH = THT(KJLE)
ISN 0282            RS = RD**EX
ISN 0283            TH = EX*TH
ISN 0284            ZOMLE = DCMPLX(RS*DCOS(TH),RS*DSIN(TH))
ISN 0285            ZOMLE = (ZD-ZB*ZOMLE/ZC)/(Z1-ZOMLE/ZC)
ISN 0286           12 IF(ITE.EQ.1) GO TO 32
ISN 0288            ZOMTE = ZD
ISN 0289            ZA(1) = ZOMTE
ISN 0290            GO TO 33
ISN 0291           32 RD=E(1)
ISN 0292            TH=THT(1)
ISN 0293            RS=RD**EX
ISN 0294            TH=EX*TH
ISN 0295            ZOMTE=DCMPLX(RS*DCOS(TH),RS*DSIN(TH))
ISN 0296            ZOMTE=(ZD-ZB*ZOMTE/ZC)/(Z1-ZOMTE/ZC)
ISN 0297            ZA(1)=ZOMTE
ISN 0298           33 DO 76 KJ = 2,KJLM
ISN 0299            K = KJLE - KJ
ISN 0300           76 ZA(KJ) = ZOMP(K)
ISN 0301            ZA(KJLE) = ZOMLE
ISN 0302            DO 77 K = 1,KJS
ISN 0303            KJ = KJLE + K
ISN 0304           77 ZA(KJ) = ZOMS(K)
ISN 0305            ZA(KJMX) = ZA(1)
ISN 0306            ZNTRD = DCMPLX(0.0D0,0.0D0)
ISN 0307            AREA = 0.0D0
ISN 0308            RMIN=CDABS(ZA(1))
ISN 0309            RMAX=RMIN
```

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ISN 0310            ZMAX=ZA(1)
ISN 0311            ZMIN=ZA(1)
ISN 0312            KJMXX = 1
ISN 0313            KJMN=1
ISN 0314            DO 78 KJ = 2,KJMX
ISN 0315            DAREA = DABS(DREAL(ZA(KJ-1))*DIMAG(ZA(KJ))-DREAL(ZA(KJ))*
                   *    DIMAG(ZA(KJ-1)))/2.0D0
ISN 0316            ZBR = (ZA(KJ-1)+ZA(KJ))/3.0D0
ISN 0317            ZNTRD = ZNTRD + ZBR*DAREA
ISN 0318            RABS=CDABS(ZA(KJ))
ISN 0319            IF(RABS.GE.RMIN) GO TO 79
ISN 0321            RMIN=RABS
ISN 0322            ZMIN=ZA(KJ)
ISN 0323            KJMN=KJ
ISN 0324            GO TO 78
ISN 0325         79 IF(RABS.LE.RMAX) GO TO 78
ISN 0327            RMAX=RABS
ISN 0328            ZMAX=ZA(KJ)
ISN 0329            KJMXX = KJ
ISN 0330         78 AREA = AREA + DAREA
ISN 0331            RATIO=RMAX/RMIN
ISN 0332            ZNTRD = ZNTRD/AREA
ISN 0333            ZOMSTR= ZC*(ZNTRD-ZD)/(ZNTRD-ZB)
ISN 0334            WRITE(6,602) ITER,ZD,ZB,ZC,ZOMSTR,ZNTRD,RMIN,RMAX,RATIO,
                   1             (ZA(KJ),KJ=1,KJMX)
ISN 0335        602 FORMAT(/I5,1P10E12.4/E17.4,2E12.4/(10E13.5))
ISN 0336            IF(RATIO.LT.RTOL) GO TO 63
ISN 0338            IF(IGOT.EQ.1) GO TO 63
ISN 0340            ITER = ITER + 1
ISN 0341            IF(ITER.LE.30) GO TO 62
ISN 0343            WRITE(6,204)
ISN 0344        204 FORMAT(///10X,'TOLERANCE SPECIFIED FOR RMAX/RMIN NOT MET IN',
                   *    ' 30 ITERATIONS')
ISN 0345            STOP
ISN 0346         62 CONTINUE
ISN 0347            IF(M.EQ.2) GO TO 66
ISN 0349            ZDS=1.1D0*ZMIN
ISN 0350            KJ=KJMN
ISN 0351         67 RD=E(KJ)**EX
ISN 0352            TH=EX*THT(KJ)
ISN 0353            ZOM=DCMPLX(RD*DCOS(TH),RD*DSIN(TH))
ISN 0354            ZC=(ZOM*(ZDS-ZG)+ZE)/(ZDS+ZF-ZE*ZOM)
ISN 0355            GO TO 60
ISN 0356         66 M=1
ISN 0357            ZDS=0.9D0*ZMAX
ISN 0358            KJ=KJMXX
ISN 0359            GO TO 67
ISN 0360         63 IGOT = 1
ISN 0361            WRITE(6,208) ZD,ZB,ZC,ZNTRD,ZOMSTR
ISN 0362        208 FORMAT(//10X,  'CONSTANTS FOR MAPPING FROM ',
                   *    'Z - PLANE TO OMEGA - PLANE ARE',
                   *    //20X,'A = ',1P2E20.5,/20X,'B = ',1P2E20.5,/20X,'C = ',
                   *    1P2E20.5,//20X,'ZNTRD = ',1P2E20.5,/20X,'ZOMSTR = ',1P2E20.5)
                C
                C  SET UP THE ARRAYS OF THETA AND LN(R)
                C
ISN 0363            DO 41 KJ = 1,KJMXM
ISN 0364            X(KJ) = DATAN2(DIMAG(ZA(KJ)),DREAL(ZA(KJ)))
```

```
ISN 0365          41 Y(KJ) = CDABS(ZA(KJ))
ISN 0366             X(KJMX) = X(1) + TPI
ISN 0367             Y(KJMX) = Y(1)
                  C
                  C  NOW ADJUST THE ARGUMENTS OF THE THETA ARRAY, SO AS TO BE CONTINUOUS
                  C  ACROSS THE BRANCH CUT (ALONG THE NEGATIVE REAL AXIS) OF THE
                  C  DATAN2 FUNCTION.  THIS ADJUSTMENT ASSUMES THAT THE CONTOUR IS
                  C  TRAVERSED IN A COUNTERCLOCKWISE DIRECTION.
                  C
ISN 0368             BR = 0.0D0
ISN 0369             PO = X(1)
ISN 0370             DO 410 KJ = 2,KJMXM
ISN 0371             IF(DABS(X(KJ)-PO).GT.PI) BR = 1.0D0
ISN 0373             PO = X(KJ)
ISN 0374             X(KJ) = X(KJ) + BR*TPI
ISN 0375        410 CONTINUE
                  C
ISN 0376             RMIN = 10.0D0
ISN 0377             RMAX = 0.0D0
ISN 0378             DO 49 K = 1,KJMX
ISN 0379             IF(Y(K).LT.RMIN) RMIN = Y(K)
ISN 0381          49 IF(Y(K).GT.RMAX) RMAX = Y(K)
ISN 0383             WARSCH = DSQRT(RMAX/RMIN) - 1.0D0
ISN 0384             IF(WARSCH.LT.0.3D0) OM = 1.0D0
ISN 0386             WRITE(6,202)
ISN 0387             WRITE(6,243)
ISN 0388        243 FORMAT(3X, 'BLADE-SURFACE IMAGE IN THE OMEGA PLANE:',
                    *    /3X,'KJ',6X,'REAL',10X,'IMAG',12X,'R',9X,'THETA',/)
ISN 0389             DO 51 KJ = 1,KJMX
ISN 0390             WRITE(6,270) KJ,ZA(KJ),Y(KJ),X(KJ)
ISN 0391          51 CONTINUE
ISN 0392             DO 43 KJ = 1,KJMX
ISN 0393          43 Y(KJ) = DLOG(Y(KJ))
                  C
                  C  USE FFT TO FIND A(N) AND B(N)
                  C
                  C  FIRST SET UP THE CONSTANTS FOR THE LN(R), THETA SPLINE FIT
                  C
ISN 0394             CALL CISPLN(Y,X,E,F,KJMX,1,128,1)
                  C
ISN 0395             ZI = DCMPLX(0.0D0,1.0D0)
ISN 0396             N = 64
ISN 0397             N2 = 128
ISN 0398             NP1 = N + 1
ISN 0399             NP2 = N + 2
ISN 0400             PBN = PI/64.0D0
ISN 0401             NB2 = N/2
ISN 0402             NB2P1 = NB2 + 1
ISN 0403             ZNN = DCMPLX(DFLOAT(N),0.0D0)
ISN 0404             ZW2N = DCMPLX(DCOS(PBN),DSIN(PBN))
ISN 0405             OMM = 1.0D0 - OM
                  C
ISN 0406             CALL THDGRK
                  C
                  C  FIND ZETAA AND ZETAB
                  C
ISN 0407             ZABST = ZMGA
ISN 0408             ZBBST = ZMGB
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ISN 0409            RABST = 1.0D0
ISN 0410            RBBST = 1.0D0
ISN 0411            DR = 0.1D0
ISN 0412            DTH = 10.0D0*PI/180.0D0
ISN 0413            THA = -6.0D0*DTH
ISN 0414            DO 21 I = 1,6
ISN 0415            R = .49D0 + DR*DFLOAT(I-1)
ISN 0416            DO 22 J = 1,13
ISN 0417            TH = DTH*DFLOAT(J-1)  + THA
ISN 0418            ZTAGS = DCMPLX(R*DCOS(TH),R*DSIN(TH))
ISN 0419            CALL OMETA(A,B,ZMG ,ZTAGS,ZTANSR,65,1.0D-00,1)
ISN 0420            RA = CDABS(ZMG-ZMGA)
ISN 0421            IF(RA.GT.RABST) GO TO 23
ISN 0423            RABST = RA
ISN 0424            ZABST = ZTAGS
ISN 0425         23 ZTAGS = -ZTAGS
ISN 0426            CALL OMETA(A,B,ZMG ,ZTAGS,ZTANSR,65,1.0D-00,1)
ISN 0427            RB = CDABS(ZMG-ZMGB)
ISN 0428            IF(RB.GT.RBBST) GO TO 22
ISN 0430            RBBST = RB
ISN 0431            ZBBST = ZTAGS
ISN 0432         22 CONTINUE
ISN 0433         21 CONTINUE
ISN 0434            ZTAGS = ZABST
ISN 0435            M = 0
ISN 0436            CALL OMETA(A,B,ZMGA,ZTAGS,ZTANSR,65,1.0D-05,M)
ISN 0437            IF(M.EQ.0) GO TO 260
ISN 0439            WRITE(6,261) ZTAGS,RABST
ISN 0440        261 FORMAT(//5X,'OMETA FAILED TO CONVERGE FOR ZETA A:',
                   *    /10X,'ZTAGS = ',1P2E13.5,'  RABST =',E13.5)
ISN 0441            STOP
ISN 0442        260 CONTINUE
ISN 0443            ZETAA = ZTANSR
ISN 0444            ZTAGS = ZBBST
ISN 0445            M = 0
ISN 0446            CALL OMETA(A,B,ZMGB,ZTAGS,ZTANSR,65,1.0D-05,M)
ISN 0447            IF(M.EQ.0) GO TO 262
ISN 0449            WRITE(6,263) ZTAGS,RBBST
ISN 0450        263 FORMAT(//5X,'OMETA FAILED TO CONVERGE FOR ZETA B:',
                   *    /10X,'ZTAGS = ',1P2E13.5,'  RBBST =',E13.5)
ISN 0451            STOP
ISN 0452        262 CONTINUE
ISN 0453            ZETAB = ZTANSR
                C
                C  FIND GAMMA, ALPHA, BETA, AND S FOR MAPPING TO ETA - PLANE
                C
ISN 0454            AP = CDABS(ZETAA + ZETAB)
ISN 0455            AM = CDABS(ZETAA - ZETAB)
ISN 0456            AB = CDABS(ZETAA*ZETAB)
ISN 0457            CHY = (2.0D0-AP*AP+2.0D0*AB*AB)/AM/AM
ISN 0458            RT = DSQRT(CHY*CHY-1.0D0)
ISN 0459            CA = DSQRT(DABS(CHY+RT))
ISN 0460            CB = DSQRT(DABS(CHY-RT))
ISN 0461            SS = DMIN1(CA,CB)
ISN 0462            ZAL = (2.0D0*ZETAA*ZETAB+(SS*SS*(ZETAA-ZETAB)-ZETAA-ZETAB)
                   *  /DCONJG(ZETAA))/(SS*SS*(ZETAA-ZETAB)+ZETAA+ZETAB-2.0D0/
                   *  DCONJG(ZETAA))
ISN 0463            ZBT = (2.0D0*ZETAA*ZETAB-ZAL*(ZETAA+ZETAB))/(ZETAA+ZETAB-2.0D0
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                     *    *ZAL)
ISN 0464                   ZGM = SS*(ZETAA-ZBT)/(ZETAA-ZAL)
                  C
ISN 0465            415 WRITE(6,245) ZABST
ISN 0466            245 FORMAT(//10X,'BEST GUESS FOR ZETA A IS ZABST = ',1P2E12.3)
ISN 0467                WRITE(6,246) ZBBST
ISN 0468            246 FORMAT(//10X,'BEST GUESS FOR ZETA B IS ZBBST = ',1P2E12.3)
ISN 0469                WRITE(6,215) ZETAA,ZETAB
ISN 0470            215 FORMAT(// 5X,'ZETAA = ',1P2E13.5,'  ZETAB = ',2E13.5)
ISN 0471                WRITE(6,216) ZAL,ZBT,ZGM,SS
ISN 0472            216 FORMAT(// 5X,'ALPHA = ',1P2E11.3,'  BETA = ',2E11.3,
                     *    '   GAMMA = ',2E11.3,'   S = ',2E11.3)
                  C
                  C  FINDING THE LOCATION OF BLADE-SURFACE POINTS IN THE ZETA PLANE ONLY
                  C  INVOLVES PHI(THETA), SINCE R = 1.  USE SPLINE INTERPOLATION
                  C
ISN 0473                PHI(129) = PHI(1) + TPI
ISN 0474                E(129) = E(1) + TPI
ISN 0475                CALL CISPLN(PHI,E,THT,F,129,1,1,2)
ISN 0476                CALL CISPLN(PHI,E,X,F,129,2,KJMX,2)
                  C
                  C  WHEN THE THETA VS. PHI CURVE IS VERY STEEP, IT MAY HAPPEN THAT THE
                  C  SPLINE-FITTED PHI VS. THETA CURVE IS NOT MONOTONIC: CHECK NOW WHETHER
                  C  THIS HAS HAPPENED, AND REPLACE THE SPLINE-FITTED DATA WITH LINEAR
                  C  INTERPOLATES WHEREVER IT HAS.
                  C
ISN 0477                DO 810 K = 2,KJMX
ISN 0478                IF(F(K).GE.F(K-1)) GO TO 810
ISN 0480                DO 811 I = 2,129
ISN 0481                IF(E(I).GT.X(K)) GO TO 812
ISN 0483            811 CONTINUE
ISN 0484                I = 129
ISN 0485            812 IP = I
ISN 0486                IM = I-1
ISN 0487                CON =(PHI(IP)-PHI(IM))/(E(IP)-E(IM))
ISN 0488                J = K
ISN 0489                DO 815 JJ = 1,20
ISN 0490                IF(X(J+JJ).GT.E(IP)) GO TO 816
ISN 0492            815 CONTINUE
ISN 0493            816 KP = J + JJ - 1
ISN 0494                DO 817 JJ = 1,20
ISN 0495                IF(X(J-JJ).LT.E(IM)) GO TO 818
ISN 0497            817 CONTINUE
ISN 0498            818 KM = J - JJ+ 1
ISN 0499                DO 819 JJ = KM,KP
ISN 0500                TMP = F(JJ)
ISN 0501                F(JJ) = PHI(IM) + CON*(X(JJ)-E(IM))
ISN 0502                WRITE(6,813) JJ,F(JJ),TMP
ISN 0503            813 FORMAT(5X,'NOTE: SPLINE FIT REPLACED BY LINEAR INTERPOLATION IN',
                     *    ' FINDING PHI(',I3,') =',F10.5,'  OLD PHI =',F10.5)
ISN 0504            819 CONTINUE
ISN 0505                K = KP
ISN 0506            810 CONTINUE
                  C
ISN 0507                WRITE(6,217)
ISN 0508            217 FORMAT(//10X,'MAPPING FROM OMEGA - PLANE TO ZETA - PLANE:',
                     *    // 4X,'K',14X,'OMEGA',26X,'ZETA',//)
ISN 0509                DO 54 K = 1,KJMX
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ISN 0510            R = DEXP(Y(K))
ISN 0511            ZX = DCMPLX(R*DCOS(X(K)),R*DSIN(X(K)))
ISN 0512            ZYG = DCMPLX(DCOS(F(K)),DSIN(F(K)))
ISN 0513            IF(K.EQ.1.OR.K.EQ.KJMX) ZYG = Z1
ISN 0515            ZCC(K) = ZYG
ISN 0516         54 WRITE(6,218) K,ZX,ZYG
ISN 0517        218 FORMAT(I5,1P4E15.5)
                C
                C  ZCC NOW CONTAINS ZETA ON THE BLADE SURFACES
                C
ISN 0518            WRITE(6,202)
                C
                C  NOW DO THE MAPPING FROM THE ETA - PLANE TO THE KSI TILDE - PLANE,
                C    WHERE ETA/S = SN(KSI TILDE)
                C
ISN 0519            AK = SS*SS
ISN 0520            AKQ = AK*AK
ISN 0521            AKP = DSQRT(1.0D0-AKQ)
ISN 0522            AKM = AKP*AKP
ISN 0523            CALL ELLPT(PI,AK,RL,1)
ISN 0524            TBK = 2.0D0*RL
ISN 0525            CTR = -RL
ISN 0526            CAPK = RL
ISN 0527            CALL ELLPT(PI,AKP,RL,1)
ISN 0528            CAPKPM = RL
ISN 0529            WRITE(6,222) AK,CAPK,AKP,CAPKPM
ISN 0530        222 FORMAT(//10X, 'COMPLETE ELLIPTIC INTEGRALS  OF K AND K PRIME',
                   *    ' ARE AS FOLLOWS:',
                   *  //10X,'K(',F10.6,') = ',F10.6,5X,'K(',F10.6,') = ',F10.6)
ISN 0531            IFIND=0
ISN 0532            DO 55 I = 1,KJMX
ISN 0533            ZA(I) = ZGM*(ZCC(I)-ZAL)/(ZCC(I)-ZBT)
                C
                C  ZA(I) NOW HOLDS ETA
                C
ISN 0534            ZTD = ZA(I)/SS
ISN 0535            TAU = DREAL(ZTD)
ISN 0536            DLT = DIMAG(ZTD)
ISN 0537            TSQ = TAU*TAU
ISN 0538            DSQ = DLT*DLT
ISN 0539            ART = 1.0D0 + AKQ*(TSQ + DSQ)
ISN 0540            RT = DSQRT((1.0D0-AKQ*TSQ)*(1.0D0-AKQ*TSQ) + AKQ*DSQ*(2.0D0*
                   *   (1.0D0+AKQ*TSQ)
                   *   +AKQ*DSQ))
ISN 0541            BRQ = 1.0D0 + TSQ + DSQ
ISN 0542            BRT = DSQRT((1.0D0-TSQ)*(1.0D0-TSQ) + DSQ*(DSQ+2.0D0+2.0D0*TSQ))
ISN 0543            ALM = (BRQ-BRT)*(ART-RT)/4.0D0/AKQ/TSQ
ISN 0544            SGA = (TSQ + DSQ -ALM)
ISN 0545            SGA = SGA/(SGA+1.0D0-ALM*AKQ*(TSQ+DSQ))
ISN 0546            IF(TAU.EQ.0.0D0) GO TO 403
ISN 0548            RTALM = DSQRT(ALM)*TAU/DABS(TAU)
ISN 0549            GO TO 404
ISN 0550        403 RTALM = 0.0D0
ISN 0551        404 SGN = 1.0D0
ISN 0552            IF(DLT.LT.0.0D0) SGN = -1.0D0
ISN 0554            RTSGA = SGN*DSQRT(SGA)
ISN 0555            RTALM = DARSIN(RTALM)
ISN 0556            RTSGA = DARSIN(RTSGA)
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```
ISN 0557            CALL ELLPT(RTALM,AK,RL,0)
ISN 0558            CALL ELLPT(RTSGA,AKP,AG,0)
ISN 0559            IF(AG.GE.0.0D0) GO TO 57
ISN 0561            AG = -AG
ISN 0562            RL =-RL - TBK
ISN 0563         57 ZA1(I) = DCMPLX(RL,AG)
ISN 0564            IF(IFIND.EQ.1) GO TO 55
ISN 0566            IF(I.EQ.1) GO TO 55
ISN 0568            IF((RL-DREAL(ZA1(I-1))).LT.0.0D0) GO TO 55
ISN 0570            KEDGE = I
ISN 0571            KGM = I - 1
ISN 0572            IFIND = 1
ISN 0573         55 CONTINUE
                 C
                 C  ZA1(I) NOW HOLDS KSI HAT
                 C
ISN 0574            WRITE(6,219)
ISN 0575        219 FORMAT(////10X,'MAPPING FROM THE ETA - PLANE TO THE KSI HAT - ',
                    *' PLANE',
                    *   /// 4X,'K',15X,'ETA',25X,'KSI HAT',//)
ISN 0576            DO 56 K = 1,KJMX
ISN 0577            WRITE(6,218) K,ZA(K),ZA1(K)
ISN 0578         56 CONTINUE
                 C
                 C  NOW SET UP A GRID IN THE KSI-HAT PLANE, AND MAP IT BACK
                 C  TO THE Z - PLANE:
                 C
ISN 0579            WRITE(6,202)
ISN 0580            WRITE(6,205)
ISN 0581        205 FORMAT(/5X,'MAPPING OF A GRID IN THE KSI-HAT PLANE',
                    *   //3X,'K   L', 8X,'KSI HAT',
                    *                          15X,'ETA',16X,'ZETA',16X,'OMEGA',
                    *   13X,'Z MAPPED',14X,'ZXY',//)
ISN 0582            ZA2(1) = ZTE
ISN 0583            ZA2(KJMX) = ZTE
ISN 0584            ZA2(KJLE) = ZLE
ISN 0585            DO 91 KJ = 2,KJLM
ISN 0586            K = KJLE - KJ
ISN 0587         91 ZA2(KJ) = ZP(K)
ISN 0588            DO 92 K = 1,KJS
ISN 0589            KJ = KJLE + K
ISN 0590         92 ZA2(KJ) = ZS(K)
                 C
                 C  THE ZA2 ARRAY NOW HOLDS THE BLADE-SURFACE COORDINATES, IN THE ORDER:
                 C    KJ = 1: TE
                 C    KJ = 2,KJLM: PRESSURE SIDE, FROM TE TO LE
                 C    KJ = KJLE: LE
                 C    KJ = KJLP,KJMXM: SUCTION SIDE, FROM LE TO TE
                 C    KJ = KJMX: TE AGAIN
                 C
ISN 0591            KMXM1 = KMX - 1
ISN 0592            LMXM1 = LMX - 1
ISN 0593            HCPKPM = CAPKPM/2.0D0
ISN 0594            TWOCPK = 2.0D0*CAPK
ISN 0595            THCPK = 3.0D0*CAPK
ISN 0596            FCPK = 4.0D0*CAPK
ISN 0597            ZAG = ZAL*ZGM
ISN 0598            EXINV = 1.0D0/EX
```

```
ISN 0599            DXIR = FCPK/DFLOAT(KMXM1)
ISN 0600            SHXTE = DREAL(ZA1(1))
ISN 0601            IF(SHXTE.GT.CTR) SHXTE = SHXTE-FCPK
ISN 0603            SHK = -CAPKPM*CAPKPM/4.0D0/(THCPK      +SHXTE)
ISN 0604            SHKINV = 1.0D0/SHK
ISN 0605            IF(ISHEAR.EQ.0) SHKINV = 0.0D0
ISN 0607            DSHX = 2.0D0/DFLOAT(KMXM1)
ISN 0608            DSHY = 1.0D0/DFLOAT(LMXM1)
ISN 0609            ZEETE = ZA(1)
ISN 0610            I0 = 0
ISN 0611            K = 1
ISN 0612            L = 1
                C
                C  K - LOOP STARTS HERE
                C
ISN 0613          760 CONTINUE
                C
                C  USE  LINEAR   INTERPOLATION TO GIVE A FIRST GUESS AT Z ON THE
                C  BLADE SURFACE
                C
ISN 0614            XIR = SHXTE+DFLOAT(K-1)*DXIR
ISN 0615            IF(XIR.LT.-THCPK) XIR = XIR + FCPK
ISN 0617            IF(XIR.GT.CAPK) XIR = XIR - FCPK
ISN 0619            DO 371 I = KEDGE,KJMX
ISN 0620            IF(DREAL(ZA1(I)).LT.XIR) GO TO 372
ISN 0622          371 CONTINUE
ISN 0623            DO 373 I = 2,KGM
ISN 0624            IF(DREAL(ZA1(I)).LT.XIR) GO TO 372
ISN 0626          373 CONTINUE
ISN 0627            I = KEDGE
ISN 0628          372 KA = I
ISN 0629            KB = I - 1
ISN 0630            XIA = DREAL(ZA1(KA))
ISN 0631            XIB = DREAL(ZA1(KB))
ISN 0632            IF(KA.NE.KEDGE) GO TO 380
ISN 0634            IF(XIA.LT.0.0D0)XIA = XIA + FCPK
ISN 0636            IF(XIB.LT.0.0D0)XIB = XIB + FCPK
ISN 0638            IF(XIR.LT.0.0D0)XIR = XIR + FCPK
ISN 0640          380 ZGSA = ((XIR-XIA)*ZA2(KB)+(XIB-XIR)*ZA2(KA))/(XIB-XIA)
                C
ISN 0641          381 ZGS=ZGSA
ISN 0642            SHX = -1.0D0+DSHX*DFLOAT(K-1)
                C
                C  L - LOOP STARTS HERE
                C
ISN 0643          770 CONTINUE
ISN 0644            SHY=DSHY*DFLOAT(L-1)
ISN 0645            XIM=HCPKPM*(1.0D0-SHY)
ISN 0646            XIR = (XIM-HCPKPM)**2
ISN 0647            XIR = SHXTE+    TWOCPK*(1.0D0+SHX)+XIR*SHKINV
ISN 0648            ZXI = DCMPLX(XIR,XIM)
                C
                C  BYPASS IMAGE CALCULATIONS FOR POINTS THAT FALL ON THE IMAGES OF
                C  PLUS OR MINUS INFINITY OR THE T.E.
                C
ISN 0649            IF(ISHEAR.EQ.0) GO TO 84
ISN 0651            IF(L.EQ.1.AND.K.EQ.1) GO TO 73
ISN 0653            IF(L.EQ.1.AND.K.EQ.KMX) GO TO 73
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ISN 0655          84 CONTINUE
ISN 0656             IF(L.EQ.LMX.AND.K.EQ.1) GO TO 74
ISN 0658             IF(L.EQ.LMX.AND.K.EQ.KMX) GO TO 74
ISN 0660             IF(IOE.EQ.0) GO TO 80
ISN 0662             IF(L.EQ.LMX.AND.K.EQ.KMXH) GO TO 81
ISN 0664             GO TO 80
                  C
                  C
                  C
ISN 0665          73 ZEETA = ZEETE
ISN 0666             ZETA = (ZBT*ZEETA-ZAG)/(ZEETA-ZGM)
ISN 0667             ZOMA = ZOMTE
ISN 0668             ZFNL = DCMPLX(SHX,SHY)
ISN 0669             ZXY = ZTE*ZGAMMA
ISN 0670             GO TO 710
ISN 0671          74 ZEETA = DCMPLX(SS,0.0D0)
ISN 0672             ZETA = ZETAA
ISN 0673             ZOMA = ZMGA
ISN 0674             ZFNL = DCMPLX(SHX,SHY)
ISN 0675             ZXY = 2.0D0*ZA(LMX-1)-ZA(LMX-2)
ISN 0676             GO TO 710
ISN 0677          81 ZEETA = DCMPLX(-SS,0.0D0)
ISN 0678             ZETA = ZETAB
ISN 0679             ZOMA = ZMGB
ISN 0680             ZXY = 2.0D0*ZA(LMX-1)-ZA(LMX-2)
ISN 0681             ZFNL = DCMPLX(SHX,SHY)
ISN 0682             GO TO 710
                  C
                  C
                  C
ISN 0683          80 CONTINUE
ISN 0684             CALL JCELFN(XIR,XIM,AKQ,AKM,RLS,AGS,1)
ISN 0685             ZEETA = SS*DCMPLX(RLS,AGS)
ISN 0686             ZETA = (ZBT*ZEETA-ZAG)/(ZEETA-ZGM)
ISN 0687             CALL OMETA(A,B,ZOMA,ZETA,ZTANSR,65,1.0D-00,1)
                  C
                  C  NOW DO THE NEWTON-RAPHSON ITERATION TO FIND Z, GIVEN ZBOMK
                  C
ISN 0688             ZBOM=ZC*(ZOMA-ZD)/(ZOMA-ZB)
ISN 0689             RAD = CDABS(ZBOM)
ISN 0690             ARG = DATAN2(DIMAG(ZBOM),DREAL(ZBOM))
ISN 0691             RADO = RAD**EXINV
ISN 0692             ARGO = EXINV*ARG
ISN 0693             ZBOMK = DCMPLX(RADO*DCOS(ARGO),RADO*DSIN(ARGO))
ISN 0694             IT=1
ISN 0695          90 Z = ZGS
ISN 0696             ZZT = (Z-ZT)/ZDN
ISN 0697             ZT1 = ZPI*ZZT
ISN 0698             ZT2 = CDSIN(ZT1)
ISN 0699             ZZN = (Z-ZN)/ZDN
ISN 0700             ZT3 = ZPI*ZZN
ISN 0701             ZT4 = CDSIN(ZT3)
ISN 0702             ZF=ZT2/ZT4-ZBOMK
ISN 0703          87 ZFPM = ZPI*CDSIN(ZTN)/ZDN/ZT4/ZT4
ISN 0704             ZNEW = Z - ZF/ZFPM
ISN 0705             IF(KNR.EQ.0) GO TO 83
ISN 0707             IF(KOUNT.EQ.0) WRITE(6,211)
ISN 0709         211 FORMAT(//5X,'DIAGNOSTIC OUTPUT OF THE NEWTON-RAPHSON ITERATIONS:',
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                *   /5X,'VARIABLES PRINTED ARE IT,ZBOMK,Z,ZNEW,ZF,ZFPM',//)
ISN 0710            WRITE(6,212) IT,ZBOMK,Z,ZNEW,ZF,ZFPM
ISN 0711        212 FORMAT(I5,1P10E12.3)
ISN 0712            KOUNT = KOUNT + 1
ISN 0713            IF(KOUNT.GT.INR) STOP
ISN 0715         83 CONTINUE
ISN 0716            IT = IT + 1
ISN 0717            ZGS = ZNEW
ISN 0718            IF(CDABS(ZNEW-Z).LT.1.0D-06) GO TO 72
ISN 0720            IF(CDABS(ZGS).GT.SIZE ) ZGS = ZGSA
ISN 0722            IF(IT.LE.50) GO TO 90
ISN 0724         82 ZGS = ZGSA
ISN 0725            ZNEW = ZERO
ISN 0726            IF(L.EQ.1) ZNEW=ZGSA
ISN 0728         72 CONTINUE
                 C
ISN 0729            ZFNL = DCMPLX(SHX,SHY)
ISN 0730            ZXY=ZGAMMA*ZNEW
                 C
                 C  CHECK POINTS ON THE PERIODIC BOUNDARY, NEAR K=1, K=KMX, AND K=KMX/2
                 C
ISN 0731            IF(ISHEAR.EQ.0) GO TO 710
ISN 0733            IF(L.NE.LMX) GO TO 710
ISN 0735            IF(K.EQ.1.OR.K.EQ.KMX) GO TO 710
ISN 0737            IF(K.GT.4) GO TO 711
ISN 0739        714 DY = DIMAG(ZXY-ZT)
ISN 0740            IF(DY.GE.0.0D0) GO TO 717
ISN 0742            ZXY = ZXY + ZI*SG
ISN 0743            GO TO 714
ISN 0744        717 IF(DY.LE.SG) GO TO 710
ISN 0746            ZXY = ZXY - ZI*SG
ISN 0747            GO TO 714
ISN 0748        711 IF(K.GE.KAA) GO TO 712
ISN 0750            GO TO 710
ISN 0751        712 IF(K.GT.KMXH) GO TO 713
ISN 0753            IF(K.EQ.KMXH.AND.IOE.EQ.1) GO TO 710
ISN 0755        718 DY = DIMAG(ZXY-ZN)
ISN 0756            IF(DY.GE.0.0D0) GO TO 719
ISN 0758            ZXY = ZXY + ZI*SG
ISN 0759            GO TO 718
ISN 0760        719 IF(DY.LE.SG) GO TO 710
ISN 0762            ZXY = ZXY - ZI*SG
ISN 0763            GO TO 718
ISN 0764        713 IF(K.GT.KBB) GO TO 715
ISN 0766        720 DY = DIMAG(ZN-ZXY)
ISN 0767            IF(DY.GE.0.0D0) GO TO 721
ISN 0769            ZXY = ZXY - ZI*SG
ISN 0770            GO TO 720
ISN 0771        721 IF(DY.LE.SG) GO TO 710
ISN 0773            ZXY = ZXY + ZI*SG
ISN 0774            GO TO 720
ISN 0775        715 IF(K.LT.KMXM4) GO TO 710
ISN 0777        722 DY = DIMAG(ZT-ZXY)
ISN 0778            IF(DY.GE.0.0D0) GO TO 723
ISN 0780            ZXY = ZXY - ZI*SG
ISN 0781            GO TO 722
ISN 0782        723 IF(DY.LE.SG) GO TO 710
ISN 0784            ZXY = ZXY - ZI*SG
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ISN 0785            GO TO 722
                C
                C
                C
ISN 0786          710 WRITE(6,224) K,L,ZXI,ZEETA,ZETA,ZOMA,ZFNL,ZXY
ISN 0787            ZA(L) = ZXY
ISN 0788            XX(K,L) = DREAL(ZXY)
ISN 0789            YY(K,L) = DIMAG(ZXY)
ISN 0790           71 CONTINUE
ISN 0791            IF(I0.EQ.1) GO TO 790
ISN 0793            L = L + 1
ISN 0794            IF(L.LE.LMX) GO TO 770
ISN 0795            WRITE(6,202)
ISN 0797            IF(PNCHZA) WRITE(7,210) (ZA(L),L=1,LMX)
ISN 0799            K = K + 1
ISN 0800            L = 1
ISN 0801            IF(K.LE.KMX) GO TO 760
ISN 0803           70 CONTINUE
                C
                C  NOW LOOK FOR CASES WHERE ZXY = ZERO, AND TRY AGAIN, USING
                C  INTERPOLATION FROM ALL NEIGHBORING POINTS
                C
ISN 0804            WRITE(6,214)
ISN 0805          214 FORMAT(//5X,'SECOND ATTEMPT TO FIND NON-CONVERGENT CASES',//)
ISN 0806            WRITE(6,205)
ISN 0807            DO 750 K = 1,KMX
ISN 0808            DO 755 L = 2,LMX
ISN 0809            IF(XX(K,L).NE.0.0D0) GO TO 755
ISN 0811            IF(YY(K,L).NE.0.0D0) GO TO 755
ISN 0813            I0 = 1
ISN 0814            L1 = L
ISN 0815            K1 = K
ISN 0816            KM = K - 1
ISN 0817            KP = K + 1
ISN 0818            IF(K1.EQ.1) KM = KMX - 1
ISN 0820            IF(K1.EQ.KMX) KP = 2
ISN 0822            LM = L - 1
ISN 0823            LP = L + 1
ISN 0824            Z11 = DCMPLX(XX(KM,LM),YY(KM,LM))
ISN 0825            Z22 = DCMPLX(XX(K1,LM),YY(K1,LM))
ISN 0826            Z33 = DCMPLX(XX(KP,LM),YY(KP,LM))
ISN 0827            Z44 = DCMPLX(XX(KM,L1),YY(KM,L1))
ISN 0828            Z55 = DCMPLX(XX(KP,L1),YY(KP,L1))
ISN 0829            IF(L1.EQ.LMX) GO TO 756
ISN 0831            Z66 = DCMPLX(XX(KM,LP),YY(KM,LP))
ISN 0832            Z77 = DCMPLX(XX(K1,LP),YY(K1,LP))
ISN 0833            Z88 = DCMPLX(XX(KP,LP),YY(KP,LP))
ISN 0834            ZGSA = (Z11+Z22+Z33+Z44+Z55+Z66+Z77+Z88)/8.0D0
ISN 0835            GO TO 381
ISN 0836          756 ZGSA = (Z11+Z22+Z33+Z44+Z55)/5.0D0
ISN 0837            GO TO 381
ISN 0838          790 CONTINUE
ISN 0839          755 CONTINUE
ISN 0840          750 CONTINUE
ISN 0841            STOP
ISN 0842          100 FORMAT(8F10.4)
ISN 0843          202 FORMAT(///)
ISN 0844          210 FORMAT(1P4E20.13)
ISN 0845          224 FORMAT(2I4,12F10.5)
ISN 0846            END
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LEVEL 21.7 ( DEC 72 )                          OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
  ISN 0002            SUBROUTINE CISPLN(Y,X,E,F,NP,IRTN,NRTN,NPD)
                C
                C  THIS SUBROUTINE FITS A CUBIC SPLINE TO A FUNCTION Y(X), DEFINED BY
                C  NP PAIRS OF POINTS.  THE SECOND DERIVATIVE OF THE FUNCTION
                C  IS PERIODIC, WITH PERIOD 2*PI.  IF NPD = 1, THE FUNCTION
                C  ITSELF IS ALSO PERIODIC, WHILE IF NPD = 2, THE FUNCTION INCREASES
                C  BY 2.*PI EVERY PERIOD: Y(X+2.*PI) = Y(X) + 2.*PI.  SOLUTIONS FOLLOW
                C  PAGES 9 - 15 OF
                C  THE THEORY OF SPLINES AND THEIR APPLICATIONS, BY J. H. AHLBERG,
                C  E. N. NILSON, AND J. L. WALSH, ACADEMIC PRESS, 1967
                C
                C
                C  NOTE THAT Y,X,E,AND F ARE, RESPECTIVELY, ORDINATE, ABSCISSA,ABSCISSA,
                C  AND ORDINATE.
                C
  ISN 0003            IMPLICIT REAL*8(A-H,O-Y),COMPLEX*16(Z)
  ISN 0004            DIMENSION Y(160),X(160),E(160),F(160),BDA(160),EM(160),H(160)
  ISN 0005            DIMENSION S(160),T(160),V(160),D(160)
  ISN 0006            DATA TPI/6.283185307179586/
                C
                C  THIS SECTION (ENTERED WHEN IRTN = 1) USES THE NP PAIRS OF INPUT
                C  COORDINATES X AND Y TO FIND THE COEFFICIENTS OF THE SPLINE FIT.
                C
                C  THESE COEFFICIENTS - HERE CALLED EM(KJ) - ARE THE SECOND DERIVATIVES
                C  OF THE FUNCTION.
                C
                C
  ISN 0007            IF(IRTN.EQ.2) GO TO 20
  ISN 0009            NPM = NP - 1
  ISN 0010            N = NP + 1
  ISN 0011            DO 1 KJ = 2,NP
  ISN 0012          1 H(KJ) = X(KJ) - X(KJ-1)
  ISN 0013            H(N) = H(2)
  ISN 0014            DO 2 KJ = 2,NP
  ISN 0015          2 BDA(KJ) = H(KJ+1)/(H(KJ)+H(KJ+1))
  ISN 0016            E(1) = 0.0D0
  ISN 0017            F(1) = 0.0D0
  ISN 0018            S(1) = 1.0D0
  ISN 0019            DO 3 KJ = 2,NPM
  ISN 0020            DN = 2.0D0 + (1.0D0 - BDA(KJ))*E(KJ-1)
  ISN 0021            E(KJ) = -BDA(KJ)/DN
  ISN 0022            D(KJ) =
                     * 6.0D0*((Y(KJ+1)-Y(KJ))/H(KJ+1)
                     *    -(Y(KJ)-Y(KJ-1))/H(KJ))/(H(KJ)+H(KJ+1))
  ISN 0023            S(KJ) = -(1.0D0-BDA(KJ))*S(KJ-1)/DN
  ISN 0024          3 F(KJ) = (D(KJ)-(1.0D0-BDA(KJ))*F(KJ-1))/DN
  ISN 0025            Y(N) = Y(2)
  ISN 0026            IF(NPD.EQ.2) Y(N) = Y(N) + TPI
  ISN 0028            D(NP) = 6.0D0*((Y(N)-Y(NP))/H(2)
                     *    -(Y(NP)-Y(NPM))/H(NP))/(H(NP)+H(2))
  ISN 0029            NPMM = NP - 2
  ISN 0030            T(NP) = 1.0D0
  ISN 0031            V(NP) = 1.0D0
  ISN 0032            DO 5 I = 1,NPMM
  ISN 0033            KJ = NP - I
  ISN 0034            T(KJ) = E(KJ)*T(KJ+1) + S(KJ)
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ISN 0035            V(KJ) = E(KJ)*V(KJ+1) + F(KJ)
ISN 0035          6 CONTINUE
ISN 0037            EM(NP) = (D(NP)-BDA(NP)*V(2)-(1.0D0-BDA(NP))*V(NPM))/
                   *    (BDA(NP)*T(2)+(1.0D0-BDA(NP))*T(NPM) + 2.0D0)
ISN 0038            DO 4 I = 1,NPM
ISN 0039            KJ = NP - I
ISN 0040          4 EM(KJ) = E(KJ)*EM(KJ+1) + F(KJ)    + S(KJ)*EM(NP)
ISN 0041            SIZE = DABS(X(NP)-X(1))/10.0D0
ISN 0042            RETURN
                  C
                  C  THIS SECTION (ENTERED WHEN IRTN = 2) RETURNS NRTN INTERPOLATED
                  C  VALUES OF THE ORDINATE F AT ASSIGNED VALUES OF THE ABSCISSA E.
                  C
ISN 0043         20 KJ = 2
ISN 0044            DO 21 J = 1,NRTN
ISN 0045            A = E(J)
ISN 0046         24 IF(A.LE.X(KJ)) GO TO 23
ISN 0048            KJ = KJ + 1
ISN 0049            IF(KJ.LE.NP ) GO TO 24
ISN 0051            DF = A - X(NP)
ISN 0052            IF(DF.GT.SIZE) WRITE(6,200) J,E(J),NP,X(NP)
ISN 0054        200 FORMAT(//10X,'WARNING - ENTRY IN CISPLN EXCEEDS END OF BASE ',
                   *    'ARRAY',/5X,'E(',I3,') = ',1PE16.8,'  EXCEEDS X(',I3,') = ',
                   *    E16.8)
ISN 0055            DF = A - X(1)
ISN 0056            IF(DF.LT.(-SIZE)) WRITE(6,201)J,E(J),X(1)
ISN 0058        201 FORMAT(//10X,'WARNING - ENTRY IN CISPLN IS LESS THAN THE FIRST',
                   *   ' BASE POINT',
                   *    /5X,'E(',I3,') = ',1PE16.8,'  IS LESS THAN X(1) = ',E16.8)
ISN 0059            KJ = NP
ISN 0060         23 DXA = X(KJ) - A
ISN 0061            DXB = A - X(KJ-1)
ISN 0062            CBA = DXA*DXA*DXA
ISN 0063            CBB = DXB*DXB*DXB
ISN 0064            F(J) = (EM(KJ-1)*CBA+EM(KJ)*CBB)/6.0D0/H(KJ)
                   *    + DXA*(Y(KJ-1)-EM(KJ-1)*H(KJ)*H(KJ)/6.0D0)/H(KJ)
                   *    + DXB*(Y(KJ)-EM(KJ)*H(KJ)*H(KJ)/6.0D0)/H(KJ)
ISN 0065         21 CONTINUE
ISN 0066            RETURN
ISN 0067            END
```

```
LEVEL 21.7 ( DEC 72 )                          OS/360  FORTRAN H

         COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                            SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
  ISN 0002            SUBROUTINE ELLPT(AR,AK,ANS,KOMP)
  ISN 0003            IMPLICIT REAL*8(A-H,O-Y),COMPLEX*16(Z)
  ISN 0004            DIMENSION SQ(12)
  ISN 0005            DATA K/1/
  ISN 0006            DATA PI/3.141592653589793/
                C
                C  THIS SUBROUTINE EVALUATES THE ELLIPTIC INTEGRAL OF THE FIRST KIND,
                C  WITH ARGUMENT AR (AN ANGLE IN RADIANS), AND PARAMETER AK (A REAL
                C  NUMBER).
                C  THIS EVALUATION USES EQ.(14) OF:  Y. L. LUKE, 'APPROXIMATIONS
                C  FOR ELLIPTIC INTEGRALS', MATH. COMP., VOL. 22 (JULY 1968), PP 627-
                C  634, WITH N = 12.
                C  KOMP = 0,1 FOR THE INCOMPLETE, COMPLETE INTEGRAL, RESPECTIVELY.
                C
  ISN 0007            IF(K.GT.1) GO TO 11
  ISN 0009            K = 2
  ISN 0010            TNP = 25.0D0
  ISN 0011            DO 10 M = 1,12
  ISN 0012            THM = PI*FLOAT(M)/TNP
  ISN 0013            S = DSIN(THM)
  ISN 0014         10 SQ(M) = S*S
  ISN 0015         11 AKK = AK*AK
  ISN 0016            SM = 0.0D0
  ISN 0017            IF(KOMP.EQ.1) GO TO 40
  ISN 0019            TN = DTAN(AR)
  ISN 0020            DO 20 M = 1,12
  ISN 0021            SG = DSQRT(1.0D0-AKK*SQ(M))
  ISN 0022            T = DATAN(SG*TN)
  ISN 0023         20 SM = SM + T/SG
  ISN 0024            ANS = (AR + 2.0D0*SM)/TNP
  ISN 0025            RETURN
  ISN 0026         40 DO 41 M = 1,12
  ISN 0027            SG = DSQRT(1.0D0-AKK*SQ(M))
  ISN 0028         41 SM = SM + 1.0D0/SG
  ISN 0029            ANS = PI*(1.0D0+2.0D0*SM)/2.0D0/TNP
  ISN 0030            RETURN
  ISN 0031            END
```

```
LEVEL 21.7 ( DEC 72 )                          OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
  ISN 0002            SUBROUTINE JCELFN(RL,AG,AKQ,AKM,RLS,AGS,M)
                    C
                    C  WHEN M.EQ.1,
                    C  THIS SUBROUTINE RETURNS THE JACOBIAN ELLIPTIC SINE
                    C  OF A COMPLEX ARGUMENT:
                    C        RLS + I*AGS = SN(RL + I*AG,K)
                    C  USING THE ARITHMETIC - GEOMETRIC MEAN FORMULA (SEE P.571, HANDBOOK
                    C  OF MATHEMATICAL FUNCTIONS, ED. BY M. ABRAMOWITZ AND I. A. STEGUN,
                    C  U.S. NATIONAL BUREAU OF STANDARDS, APPLIED MATHEMATICS SERIES, 55,
                    C  JUNE 1964) AND THE ADDITION FORMULA FOR THE SN (SEE EQUATION 125.01
                    C  P. 24, OF HANDBOOK OF ELLIPTIC INTEGRALS FOR ENGINEERS AND
                    C  PHYSICISTS, BY P. F. BYRD AND M. D. FRIEDMAN, SPRINGER VERLAG, 1954).
                    C  AKQ = K**2, AKM = 1. - K**2
                    C
                    C  WHEN M.EQ.2, THE QUANTITIES RETURNED ARE THE REAL AND IMAGINARY PARTS
                    C  OF THE PRODUCT CN(..)*DN(..), WHICH IS THE DERIVATIVE OF THE SN(..)
                    C
  ISN 0003            IMPLICIT REAL*8(A-H,O-Y),COMPLEX*16(Z)
  ISN 0004            DIMENSION A(20),B(20),C(20),PH(20)
  ISN 0005            K = 1
  ISN 0006            A(1) = 1.0D0
  ISN 0007            B(1) = DSQRT(AKM)
  ISN 0008            C(1) = DSQRT(AKQ)
  ISN 0009          5 DO 6 I = 2,20
  ISN 0010            A(I) = (A(I-1)+B(I-1))/2.0D0
  ISN 0011            B(I) =DSQRT(A(I-1)*B(I-1))
  ISN 0012            C(I) = (A(I-1)-B(I-1))/2.0D0
  ISN 0013            IF(DABS(C(I)).LT.1.0D-09) GO TO 7
  ISN 0015          6 CONTINUE
  ISN 0016            WRITE(6,200) RL,AG
  ISN 0017        200 FORMAT(///10X,'JCELFN FAILED TO CONVERGE FOR Z = ',1P2E15.4)
  ISN 0018            STOP
  ISN 0019          7 NM = I-1
  ISN 0020            N = I
  ISN 0021            IF(K.EQ.2) GO TO 20
  ISN 0023            PH(N) = A(N)*RL*2**NM
  ISN 0024         15 DO 11 L = 1,NM
  ISN 0025            J = N - L
  ISN 0026         11 PH(J) = (PH(J+1)+DARSIN(C(J+1)*DSIN(PH(J+1))/A(J+1)))/2.0D0
  ISN 0027            IF(K.EQ.2) GO TO 40
  ISN 0029            SNK = DSIN(PH(1))
  ISN 0030            CNK = DCOS(PH(1))
  ISN 0031            DNK = CNK/DCOS(PH(2)-PH(1))
  ISN 0032            K = 2
  ISN 0033            TMP = B(1)
  ISN 0034            B(1) = C(1)
  ISN 0035            C(1) = TMP
  ISN 0036            GO TO 5
  ISN 0037         20 PH(N) = A(N)*AG*2**NM
  ISN 0038            GO TO 15
  ISN 0039         40 SNP = DSIN(PH(1))
  ISN 0040            CNP = DCOS(PH(1))
  ISN 0041            DNP = CNP/DCOS(PH(2)-PH(1))
  ISN 0042            DNM = 1.0D0 - SNP*SNP*DNK*DNK
  ISN 0043            IF(M.EQ.2) GO TO 50
  ISN 0045            RLS = SNK*DNP/DNM
```

```
ISN 0046            AGS = CNK*DNK*SNP*CNP/DNM
ISN 0047            RETURN
ISN 0048         50 RLA = CNK*CNP
ISN 0049            AGA = SNK*DNK*SNP*DNP
ISN 0050            RLB = DNK*CNP*DNP
ISN 0051            AGB = AKQ*SNK*CNK*SNP
ISN 0052            RLS = (RLA*RLB-AGA*AGB)/DNM/DNM
ISN 0053            AGS = (-RLA*AGB-RLB*AGA)/DNM/DNM
ISN 0054            RETURN
ISN 0055            END
```

```
LEVEL 21.7 ( DEC 72 )                          OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
  ISN 0002            SUBROUTINE OMETA(A,B,ZMGA,ZTAGS,ZTANSR,N,EPS,M)
  ISN 0003            IMPLICIT REAL*8(A-H,O-Y),COMPLEX*16(Z)
  ISN 0004            DIMENSION A(65),B(65),ZC(65)
                C
                C  IF M.EQ.0,
                C  THIS SUBROUTINE USES NEWTON - RAPHSON TO FIND ZETA(OMEGA):  ZMGA IS A
                C  KNOWN VALUE OF OMEGA, ZTAGS IS THE INITIAL GUESS AT ZETA, ZTANSR IS
                C  THE SOLUTION, AND EPS IS THE TOLERANCE ON THE ANSWER.
                C
                C  IF   M.EQ.1., THIS SUBROUTINE RETURNS THE VALUE OF OMEGA (IN ZMGA)
                C  FOR A GIVEN VALUE OF ZETA (IN ZTAGS).
                C
                C  IF M.EQ.2, THE QUANTITY RETURNED (IN ZTANSR) IS D OMEGA/D ZETA, FOR
                C  GIVEN VALUES OF OMEGA (IN ZMGA) AND ZETA (IN ZTAGS)
                C
  ISN 0005            Z1 = DCMPLX(1.0D0,0.0D0)
  ISN 0006            ZETA = ZTAGS
  ISN 0007            NM = N - 1
  ISN 0008            IT = 1
  ISN 0009          5 CONTINUE
  ISN 0010            ZSMA = DCMPLX(A(N),B(N))
  ISN 0011            IF(M.EQ.1) GO TO 22
  ISN 0013            ZSMB = ZSMA*DFLOAT(NM)
  ISN 0014            IF(M.EQ.2) GO TO 40
  ISN 0016            DO 10 J = 1,NM
  ISN 0017            ZSMA = ZETA*ZSMA + DCMPLX(A(N-J),B(N-J))
  ISN 0018            ZSMB = ZETA*ZSMB + DCMPLX(A(N-J),B(N-J))*DFLOAT(NM-J)
  ISN 0019         10 CONTINUE
  ISN 0020            ZEXP = CDEXP(ZSMA)
  ISN 0021            ZG = ZETA*ZEXP - ZMGA
  ISN 0022            ZDG = ZEXP*(Z1 + ZSMB)
  ISN 0023            ZETOLD = ZETA
  ISN 0024            ZETA = ZETOLD - ZG/ZDG
  ISN 0025            IF(CDABS(ZETA-ZETOLD).LT.EPS) GO TO 20
  ISN 0027            IT = IT + 1
  ISN 0028            IF(CDABS(ZETA).GT.1.0D0) ZETA = 0.9D0*ZETA/CDABS(ZETA)
  ISN 0030            IF(IT.LE.50) GO TO 5
  ISN 0032            M = 5
  ISN 0033            RETURN
  ISN 0034         20 ZTANSR = ZETA
  ISN 0035            RETURN
  ISN 0036         22 DO 21 J = 1,NM
  ISN 0037            ZSMA = ZETA*ZSMA + DCMPLX(A(N-J),B(N-J))
  ISN 0038         21 CONTINUE
  ISN 0039            ZEXP = CDEXP(ZSMA)
  ISN 0040            ZMGA = ZETA*ZEXP
  ISN 0041            RETURN
  ISN 0042         40 DO 41 J = 1,NM
  ISN 0043         41 ZSMB = ZETA*ZSMB + DCMPLX(A(N-J),B(N-J))*DFLOAT(NM-J)
  ISN 0044            ZTANSR = ZMGA*(Z1+ZSMB)/ZTAGS
  ISN 0045            RETURN
  ISN 0046            END
```

```
LEVEL 21.7 ( DEC 72 )                          OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
  ISN 0002            SUBROUTINE SHAPE(C,H,G,EX,ZP,ZS,KJS,KJP)
  ISN 0003            IMPLICIT REAL*8(A-H,O-Y),COMPLEX*16(Z)
  ISN 0004            DIMENSION ZS(80),ZP(80)
  ISN 0005            DO 10 K = 1,KJS
  ISN 0006         10 READ(5,100) ZS(K)
  ISN 0007            DO 20 K = 1,KJP
  ISN 0008         20 READ(5,100) ZP(K)
  ISN 0009        100 FORMAT(8F10.0)
  ISN 0010            RETURN
  ISN 0011            END
```

```
LEVEL 21.7 ( DEC 72 )                          OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
  ISN 0002            SUBROUTINE SHUFL(N,ZA,ZCC)
                      C
                      C  THIS SUBROUTINE TAKES THE N COMPLEX VALUES IN ARRAY ZA, WHICH WERE
                      C  COMPUTED AND STORED IN REVERSE BINARY ORDER BY FFT2 AND "SHUFFLES"
                      C  THEM INTO PROPER ORDER USING ARRAY ZCC FOR INTERMEDIATE STORAGE.
                      C  N IS ASSUMED TO HAVE THE FORM 2**M.
                      C
  ISN 0003            IMPLICIT REAL*8(A-H,O-Y),COMPLEX*16(Z)
  ISN 0004            DIMENSION ZA(65),ZCC(65),IAL(6),KR(64)
  ISN 0005            DATA KALL/0/
  ISN 0006            DATA IAL/6*0/
  ISN 0007            IF (KALL.EQ.1) GO TO 10
  ISN 0009            KALL = 1
  ISN 0010            DO 341 JP = 1,N
  ISN 0011            J = JP - 1
  ISN 0012            IAL(6) = J/32
  ISN 0013            J = J - 32*IAL(6)
  ISN 0014            IAL(5) = J/16
  ISN 0015            J = J - 16*IAL(5)
  ISN 0016            IAL(4) = J/8
  ISN 0017            J = J - 8*IAL(4)
  ISN 0018            IAL(3) = J/4
  ISN 0019            J = J - 4*IAL(3)
  ISN 0020            IAL(2) = J/2
  ISN 0021            J = J - 2*IAL(2)
  ISN 0022            IAL(1) = J
  ISN 0023        341 KR(JP) =
                     *      32*IAL(1)+16*IAL(2)+8*IAL(3)+4*IAL(4)+2*IAL(5)+IAL(6)
  ISN 0024         10 DO 342 J = 1,N
  ISN 0025        342 ZCC(J) = ZA(KR(J)+1)
  ISN 0026            DO 360 JP = 1,N
  ISN 0027        360 ZA(JP) = ZCC(JP)
  ISN 0028            RETURN
  ISN 0029            END
```

```
LEVEL 21.7 ( DEC 72 )                          OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
  ISN 0002            SUBROUTINE THDGRK
                     C
                     C  THIS SUBROUTINE MAPS AN OVAL TO A UNIT CIRCLE, USING A VARIANT OF
                     C  THE THEODORSEN-GARRICK TRANSFORMATION AND FAST FOURIER TRANSFORM
                     C  TECHNIQUES.  (SEE REFERENCES AT BEGINNING OF MAIN PROGRAM).
                     C
  ISN 0003            IMPLICIT REAL*8(A-H,O-Y),COMPLEX*16(Z)
  ISN 0004            COMMON/TGINTG/N,NP1,NP2,N2,NB2,NB2P1,IP,IPMX,ITP,IWK,IMX,KJMX
  ISN 0005            COMMON/TGCMPX/Z1,ZW2N,ZI,ZNN,ZA,ZCC,ZA1,ZA2
  ISN 0006            COMMON/TGDBLE/PBN,OM,OMM,ANGERR,A,B,E,F,THT,PHI,X,Y
  ISN 0007            DIMENSION X(160),Y(160),E(150),F(150),THT(160)
  ISN 0008            DIMENSION PHI(130),A(65),B(65),ZCC(65),
                     *   ZA(165),ZA1(165),ZA2(165),
                     *           IWK(7)
  ISN 0009            DIMENSION ITP(100)
                     C
  ISN 0010          301 DO 300 I = 1,N2
  ISN 0011          300 PHI(I)=PBN*DFLOAT(I-1)
                     C
                     C  THE FOLLOWING CALSPAN LIBRARY ROUTINE PLACES A ZERO IN THE LOCATIONS
                     C  FROM THE FIRST TO THE LAST ARGUMENT OF THE CALL. THE THIRD ARGUMENT
                     C  GIVES THE LENGTH SPECIFICATION OF EACH ENTRY.
                     C
  ISN 0012            CALL CLEAR(A(1),A(65),8)
  ISN 0013            CALL CLEAR(B(1),B(65),8)
  ISN 0014            CALL CLEAR(E(1),E(150),8)
  ISN 0015            CALL CLEAR(ZCC(1),ZCC(65),16)
                     C
  ISN 0016            WRITE(6,401)
  ISN 0017          401 FORMAT(//10X,'PROGRESS OF PHI / THETA ITERATIONS IS AS FOLLOWS:'//
                       1 3X,'IT',4X,'DEMX',4X,'NO. OF THETA REVERSALS')
                     C
  ISN 0018            IT = 1
                     C
                     C  FIRST GUESS IS THETA - THETA(TRAILING EDGE) = PHI
                     C
  ISN 0019            DO 315 K = 1,N2
  ISN 0020          315 E(K) = X(1) + PHI(K)
  ISN 0021          305 DEMX = 0.0D0
  ISN 0022            B(1) = 0.0D0
                     C
                     C  USE AJ AND BJ TO GET NEXT APPROXIMATION TO THETA
                     C
  ISN 0023            ZCC(1) = DCMPLX(B(1),0.0D0)
  ISN 0024            DO 302 JP = 2,N
  ISN 0025          302 ZCC(JP) = DCMPLX(B(JP)/2.0D0,-A(JP)/2.0D0)
  ISN 0026            ZCC(NP1) = DCMPLX(0.0D0,0.0D0)
  ISN 0027            ZW = Z1/ZW2N
  ISN 0028            DO 303 NP = 1,NB2P1
  ISN 0029            ZA1(NP) = ZCC(NP) + DCONJG(ZCC(NP2-NP))
  ISN 0030            ZW = ZW*ZW2N
  ISN 0031          303 ZA2(NP) = ZW*(ZCC(NP)-DCONJG(ZCC(NP2-NP)))
  ISN 0032            DO 304 NP = 1,NB2P1
  ISN 0033          304 ZA(NP) = ZA1(NP) + ZI*ZA2(NP)
  ISN 0034            DO 309 NP = 2,NB2
  ISN 0035          309 ZA(NP2-NP) =
```

```
                *   DCONJG(ZA1(NP)) + ZI*DCONJG(ZA2(NP))
ISN 0036            CALL FFT2(ZA,6,IWK)
ISN 0037            CALL SHUFL(N,ZA,ZCC)
ISN 0038            B(1) = X(1) - PHI(1) - DREAL(ZA(1))
ISN 0039            DO 306 K = 1,64
ISN 0040            TMP = E(2*K-1)
ISN 0041            E(2*K-1) = DREAL(ZA(K)) + PHI(2*K-1)    + B(1)
ISN 0042            THT(2*K-1) = E(2*K-1)
ISN 0043            DEM = DABS(TMP-E(2*K-1))
ISN 0044            IF(DEM.GT.DEMX) DEMX = DEM
ISN 0046            E(2*K-1) = OM*E(2*K-1) + OMM*TMP
ISN 0047            TMP = E(2*K)
ISN 0048            E(2*K) = DIMAG(ZA(K)) + PHI(2*K)     + B(1)
ISN 0049            THT(2*K) = E(2*K)
ISN 0050            DEM = DABS(TMP-E(2*K))
ISN 0051            IF(DEM.GT.DEMX) DEMX = DEM
ISN 0053            E(2*K) = OM*E(2*K) + OMM*TMP
ISN 0054          306 CONTINUE
ISN 0055            IF(IT.NE.ITP(IP)) GO TO 840
ISN 0057            IP = IP + 1
ISN 0058            WRITE(6,841) IT
ISN 0059          841 FORMAT(/3X,
                *           'THETA BEFORE AND AFTER RELAXATION AT IT = ',I4)
ISN 0060            WRITE(6,832)(THT(I),I=1,128)
ISN 0061            WRITE(6,832)(E(I),I=1,128)
ISN 0062          832 FORMAT(1P10E13.5)
ISN 0063          840 CONTINUE
ISN 0064            IF(IT.GE.ITP(IPMX)) STOP
                C
                C  NOW USE THESE THETAS TO GET THE NEXT APPROXIMATION TO LN R(K)
                C
ISN 0066            CALL CISPLN(Y,X,E,F,KJMX,2,128,1)
                C
                C  NOW FIND THE AJ AND BJ COEFFICIENTS CORRESPONDING TO THE LN R(K) DATA
                C
ISN 0067            DO 310 JP = 1,N
ISN 0068          310 ZA(JP) = DCMPLX(F(2*JP-1),F(2*JP))
ISN 0069            DO 307 NP = 1,N
ISN 0070          307 ZA(NP) = DCONJG(ZA(NP))
ISN 0071            CALL FFT2(ZA,6,IWK)
ISN 0072            DO 308 JP = 1,N
ISN 0073          308 ZA(JP) = DCONJG(ZA(JP))/ZNN
ISN 0074            CALL SHUFL(N,ZA,ZCC)
ISN 0075            ZA(65) = ZA(1)
ISN 0076            DO 311 NP = 1,NB2P1
ISN 0077            ZA1(NP) = (DCONJG(ZA(NP2-NP)) + ZA(NP))/2.0D0
ISN 0078          311 ZA2(NP) = ZI*(DCONJG(ZA(NP2-NP)) - ZA(NP))/2.0D0
ISN 0079            ZW = ZW2N
ISN 0080            DO 312 NP = 1,NB2P1
ISN 0081            ZW = ZW/ZW2N
ISN 0082          312 ZCC(NP) = (ZA1(NP)+ZA2(NP)*ZW)/2.0D0
ISN 0083            ZW = Z1/ZW
ISN 0084            DO 313 I = 2,NB2
ISN 0085            ZW = ZW*ZW2N
ISN 0086            NP = NB2 + I
ISN 0087            ZBB = (ZA1(NP2-NP)+ZA2(NP2-NP)*ZW)/2.0D0
ISN 0088          313 ZCC(NP) = DCONJG(ZBB)
ISN 0089            A(1) =   DREAL(ZCC(1))
```

```
ISN 0090            ZCC(NP1)=0.5D0*DCONJG(ZA1(1)-ZA2(1))
ISN 0091            A(65) = DREAL(ZCC(65))
ISN 0092            DO 314 NP = 2,N
ISN 0093            A(NP) = 2.0D0*DREAL(ZCC(NP))
ISN 0094          314 B(NP) = 2.0D0*DIMAG(ZCC(NP))
                  C
                  C  CHECK FOR CONVERGENCE
                  C
ISN 0095            IF(IT.EQ.1) DEMX = 1.0D0
ISN 0097            NRV = 0
ISN 0098            DO 809 LL = 2,128
ISN 0099          809 IF(E(LL).LE.E(LL-1)) NRV = NRV + 1
ISN 0101            WRITE(6,400) IT,DEMX,NRV
ISN 0102          400 FORMAT(I5,1PE12.3,I8)
ISN 0103            IF(DEMX.LT.ANGERR ) GO TO 316
ISN 0105            IT = IT + 1
ISN 0106            IF(IT.LE.IMX) GO TO 305
ISN 0108            WRITE(6,213)
ISN 0109          213 FORMAT(//5X,'ITERATIONS FOR A AND B DID NOT CONVERGE')
ISN 0110            STOP
                  C
ISN 0111          316 WRITE(6,203) IT,OM,DEMX
ISN 0112          203 FORMAT(//10X,'A AND B ITERATIONS CONVERGED AT IT = ',I3,
                     *    '   USING OM = ',F6.3,'  .  MAXIMUM ANGULAR ERROR =',
                     *   1PE12.3,'  RADIANS')
ISN 0113            WRITE(6,220)
ISN 0114          220 FORMAT(// 4X,'K',11X,'THETA',15X,'PHI',//)
ISN 0115            DO 69 K = 1,128
ISN 0116           69 WRITE(6,221) K,E(K),PHI(K)
ISN 0117          221 FORMAT(I5,1P2E20.6)
                  C
ISN 0118            RETURN
ISN 0119            END
```

Appendix C

DICTIONARY OF VARIABLES

FORTRAN SYMBOL	ALGEBRAIC EQUIVALENT	DEFINITION, USE, COMMENTS
A(J), B(J)	A_j , B_j	Eq. 2-20, 2-21
AK	$k = S^2$	Eq. 2-23
AKP	$k' = \sqrt{1-k^2}$	Eq. 2-30
AKQ	k^2	-
ALM	λ	Eq. 2-31
CAPK	$K(k)$	Eq. 2-34
CAPKPM	$K'(k')$	Eq. 2-34
DLT	δ	Eq. 2-28
DXIM, DXIR	$Re(\Delta\hat{\xi}), Im(\Delta\hat{\xi})$	Eq. 2-38, 4-4
EX	$1/(2 - \frac{\tau}{\pi})$	See Figure 2
EXINV	$2 - \frac{\tau}{\pi}$	-
G, H	G , H	See Figure 1
HCPKPM	$\frac{1}{2}\ K'(k')$	-
OM	—	Relaxation factor used in ϕ , θ mapping
PBN	$\frac{\pi}{N}$	-
PHI	ϕ	-
SGA	σ	Eq. 2-31
SS	S	Eqs. 2-23
TAU	τ	Eq. 2-28

FORTRAN SYMBOL	ALGEBRAIC EQUIVALENT	DEFINITION, USE, COMMENTS
TPI	2π	-
XIR, XIM	$Re(\hat{\xi})$, $Im(\hat{\xi})$	-
ZA(N)	$A(n)$	Eq. A-9; used elsewhere for temporary storage
ZB	b	Eq. 2-9
ZC	c	Eq. 2-9
ZD	a	Eq. 2-9
ZI	$\sqrt{-1}$	-
ZN, ZT, ZLE, ZTE	z_N, z_T	See Figure 1
ZAL, ZBT, ZGM	α, β, γ	Eq. 2-22
ZA1(N), ZA2(N), ZCC(N)	$A_1(n)$, $A_2(n)$, $C(n)$	See, for example, Eq. A-9. Also used for temporary storage
ZBOM	Ω	-
ZBOMK	Ω^K	
ZEETA	η	-
ZETA	ζ	-
ZNTRD		centroid of the ω-plane
ZØMSTR		Ω - plane image of ZNTRO
ZOMS(K), ZOMP(K)	ω_s, ω_p	-
ZPLUS, ZMINUS	Ω^+, Ω^-	Eq. 1-6
ZXI	$\hat{\xi}$	-
ZXY		$x+iy$

Appendix D

LISTING OF METRIC GENERATOR PROGRAM

```
LEVEL 21.7 ( DEC 72 )                          OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
                C
                C  PROGRAM CIMTVL - A PROGRAM TO CALCULATE THE METRICS OF A COORDINATE
                C  TRANSFORMATION FOR TURBOMACHINERY CASCADES.  FOR DOCUMENTATION,SEE:
                C
                C       J. P. NENNI AND W. J. RAE, EXPERIENCE WITH THE DEVELOPMENT OF
                C       AN EULER CODE FOR ROTOR ROWS, ASME PAPER 83-GT-36, MARCH 1983
                C       W. J. RAE, A COMPUTER PROGRAM FOR THE IVES TRANSFORMATION IN
                C       TURBOMACHINERY CASCADES, AFOSR TR-81-0154, ADA096416, NOV 1980
                C       W. J. RAE, MODIFICATIONS OF THE IVES-LIUTERMOZA CONFORMAL-
                C       MAPPING PROCEDURE FOR TURBOMACHINERY CASCADES, ASME PAPER
                C       83-GT-116, MARCH 1983
                C       W.J. RAE, REVISED COMPUTER PROGRAM FOR EVALUATING THE IVES
                C       TRANSFORMATION IN TURBOMACHINERY CASCADES, CALSPAN CORPORATION
                C       REPORT 7177-A-1, JULY 1983
                C
                C  THE PROGRAM READS A CARD DECK CONTAINING THE COORDINATES X(K,L) ,
                C  Y(K,L) ; K = 1,KMX ; L = 1,LMX, AND FINDS BY FINITE DIFFERENCES
                C  THE METRICS OF A TRANSFORMATION TO A RECTANGLE(KSI,ETA).  IN THIS
                C  RECTANGLE, THE IMAGE OF THE BLADE SURFACE LIES ALONG ONE SIDE, AND
                C  THE IMAGES OF THE POINTS AT INFINITY LIE AT CORNERS OF THE RECTANGLE
                C  (SEE THE REFERENCES ABOVE). THE METRICS ARE WRITTEN ON TAPE 1.
                C
 ISN 0002             IMPLICIT REAL*8(A-H,O-Z)
 ISN 0003             DIMENSION Q(410,6)
 ISN 0004             READ(5,100) KMX,LMX
 ISN 0005           100 FORMAT(2014)
 ISN 0006             READ(5,101) SG
 ISN 0007           101 FORMAT(8F10.4)
                C
                C  SG IS THE SLANT GAP BETWEEN BLADES
                C
 ISN 0008             DO 10 K = 1,KMX
 ISN 0009             KM1LMX = (K-1)*LMX
 ISN 0010             LKA = KM1LMX + 1
 ISN 0011             LKB = KM1LMX + LMX
 ISN 0012            10 READ(5,200)((Q(LK,5),Q(LK,6)),LK=LKA,LKB)
 ISN 0013           200 FORMAT(1P4E20.13)
                C
                C  Q(LK,5) CONTAINS X(K,L), Q(LK,6) CONTAINS Y(K,L)
                C
 ISN 0014             KM=KMX-1
 ISN 0015             LM=LMX-1
                C
                C --- SET X AND Y AT K=KMX AND L=1
                C
 ISN 0016             LK=KM*LMX+1
 ISN 0017             Q(LK,5)=Q(1,5)
 ISN 0018             Q(LK,6)=Q(1,6)
                C
                C --- FIND X AND Y AT K=1 AND L=LMX
                C
 ISN 0019             Q(LMX,5)=2D0*Q(LM,5)-Q(LMX-2,5)
 ISN 0020             Q(LMX,6)=2D0*Q(LM,6)-Q(LMX-2,6)
                C
                C --- FIND X AND Y AT K=KMX AND L=LMX
                C
```

```
ISN 0021            LK=KMX*LMX
ISN 0022            Q(LK,5)=2D0*Q(LK-1,5)-Q(LK-2,5)
ISN 0023            Q(LK,6)=2D0*Q(LK-1,6)-Q(LK-2,6)
ISN 0024            TDELXI=4D0/DFLOAT(KM)
ISN 0025            TDELZT=2D0/DFLOAT(LM)
                C
ISN 0026            DO 520 K=1,KMX
ISN 0027            KM1LMX=(K-1)*LMX
                C
ISN 0028            DO 510 L=1,LMX
ISN 0029            LK=KM1LMX+L
ISN 0030            LP1=LK+1
ISN 0031            LM1=LK-1
ISN 0032            KP1=LK+LMX
ISN 0033            KM1=LK-LMX
                C
ISN 0034            IF(K.EQ.1) GO TO 501
ISN 0036            IF(K.EQ.KMX) GO TO 501
ISN 0038            IF(L.EQ.1) GO TO 504
ISN 0040            IF(L.EQ.LMX) GO TO 505
                C
                C --- K=2 TO KM AND L=2 TO LM    (CENTERED DIFFERENCES)
                C
ISN 0042            DXDXI=(Q(KP1,5)-Q(KM1,5))/TDELXI
ISN 0043            DYDXI=(Q(KP1,6)-Q(KM1,6))/TDELXI
ISN 0044            DXDZT=(Q(LP1,5)-Q(LM1,5))/TDELZT
ISN 0045            DYDZT=(Q(LP1,6)-Q(LM1,6))/TDELZT
ISN 0046            GO TO 509
                C
ISN 0047          501 IF(L.EQ.1) GO TO 510
ISN 0049            IF(L.EQ.LMX) GO TO 510
                C
                C --- K=1 OR KMX AND L=2 TO LM
                C
ISN 0051            LK2=LMX+L
ISN 0052            LKKM=(KM-1)*LMX+L
ISN 0053            DXDXI=(Q(LK2,5)-Q(LKKM,5))/TDELXI
ISN 0054            DYDXI=(Q(LK2,6)-Q(LKKM,6))/TDELXI
ISN 0055            DXDZT=(Q(LP1,5)-Q(LM1,5))/TDELZT
ISN 0056            DYDZT=(Q(LP1,6)-Q(LM1,6))/TDELZT
ISN 0057            GO TO 509
                C
                C --- L=1 AND K=2 TO KM
                C
ISN 0058          504 LK1=KM1LMX+1
ISN 0059            LK2=LK1+1
ISN 0060            LK3=LK2+1
ISN 0061            DXDZT=(-3D0*Q(LK1,5)+4D0*Q(LK2,5)-Q(LK3,5))/TDELZT
ISN 0062            DYDZT=(-3D0*Q(LK1,6)+4D0*Q(LK2,6)-Q(LK3,6))/TDELZT
ISN 0063            GO TO 506
                C
                C --- L=LMX AND K=2 TO KM
                C
ISN 0064          505 LKS=(KMX-K)*LMX+LM
ISN 0065            LKB=KM1LMX+LM
ISN 0066            DXDZT=(Q(LKS,5)-Q(LKB,5))/TDELZT
ISN 0067            DQ=SG
ISN 0068            IF(K.GT.KMXH) DQ=-SG
```

```
ISN 0070            DYDZT=(Q(LKS,6)+DQ-Q(LKB,6))/TDELZT
                C
ISN 0071          506 DXDXI=(Q(KP1,5)-Q(KM1,5))/TDELXI
ISN 0072            DYDXI=(Q(KP1,6)-Q(KM1,6))/TDELXI
                C
ISN 0073          509 RDM1=(DXDXI*DYDZT-DXDZT*DYDXI)
                C
                C  STORE DERIVATIVES IN THE ORDER DKSI/DX,DKSI/DY,DETA/DX,DETA/DY
                C
ISN 0074            Q(LK,1)=DYDZT/RDM1
ISN 0075            Q(LK,2)=-DXDZT/RDM1
ISN 0076            Q(LK,3)=-DYDXI/RDM1
ISN 0077            Q(LK,4)= DXDXI/RDM1
ISN 0078          510 CONTINUE
ISN 0079          520 CONTINUE
                C
                C  STORE DERIVATIVES ON TAPE 1
                C
ISN 0080            LKMX=LMX*KMX
                C   WRITE(1) KMX,LMX,LKMX,((Q(I,J),I=1,LKMX),J=1,4)
                C
                C
ISN 0087            STOP
ISN 0088            END
```

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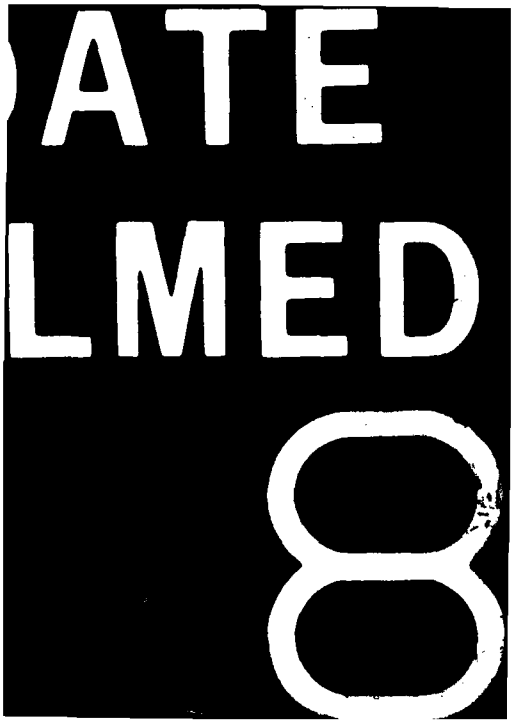
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